# Credit Traps*

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Abstract

This paper studies the limitations of monetary policy in stimulating credit and investment. We show that, under certain circumstances, unconventional monetary policies fail in that liquidity injections by the central bank into the banking sector are hoarded and not lent out. We use the term 'credit traps' to describe such situations and show how they can arise due to the interplay between financing frictions, liquidity, and collateral values. Our analysis offers a characterization of the problems created by credit traps as well as potential solutions and policy implications. Among these, the analysis shows how quantitative easing may be useful in increasing bank lending. The model further shows how small contractions in monetary policy or in loan supply can lead to collapses in lending, aggregate investment, and collateral prices.

# Introduction

Can the Federal Reserve stimulate lending when there are disruptions in the financial system? According to the credit channel literature, expansionary monetary policy alleviates financial frictions and increases the availability of credit (e.g. Bernanke and Gertler (1989, 1990, 1995)).[1] Indeed, the severity of the recent financial crisis has led central banks around the world to adopt traditional as well as unconventional policy measures to combat the crisis and boost lending (Gertler and Kiyotaki (2010)). In the U.S., the Federal Reserve experimented with new policies of quantitative and credit easing by lending directly to financial institutions, providing liquidity to key credit markets, and purchasing long term securities (Bernanke (2009)). Other major central banks such as the European Central Bank and the Bank of England followed suit with similar 'quantitative easing' policies. While there is some evidence suggesting that in the U.S. these polices have been effective in reducing credit spreads, lending by U.S. banks did not return to its pre-crisis-levels.

We study the limitations of unconventional monetary policy in stimulating credit and lending. Using a general equilibrium model with endogenous collateral values, we show that banks may rationally choose to hoard liquidity during monetary expansions rather than lend it out. Despite the best efforts of the central bank to stimulate lending, liquidity remains trapped in banks. In equilibrium, investment levels do not rise, collateral values remain depressed, and liquidity in the corporate sector remains low. We use the term 'credit traps' to describe these scenarios, and show how they can arise due to an adverse interplay between liquidity and the value of collateral.

Our model has two building blocks. The first is the well known notion that collateral eases financial frictions and increases debt capacity (see e.g. Hart and Moore (1994, 1998)). The second building block of the model is that the value of firms' collateral is determined, in part, by the liquidity constraints of industry peers. As in Shleifer and Vishny (1992) and Kiyotaki and Moore (1997), we assume that banks cannot operate assets on their own to generate cash flow and so must sell seized collateral to other industry participants. Liquidity constraints in these peer firms, therefore, affect collateral values through their impact on the amount which potential purchasers can pay for assets. In particular, when industry financial conditions are poor, the liquidation value of collateral – which is the relevant value to the bank – might be lower than the intrinsic value of

[1] Classic studies in the credit channel literature include Bernanke and Blinder (1988), Bolton and Freixas (2006), Gertler and Hubbard (1989), Gertler and Gilchrist (1994), Kashyap and Stein (1994, 1995, 2000), Lamont et al. (1994), Stein (1998), Holmström and Tirole (1997), and Allen and Gale (2000).

the assets.[2]

Based on these two building blocks, and similar to the financial accelerator literature (Bernanke and Gertler (1989), Kiyotaki and Moore (1997)), our model hinges on a feedback loop between collateral values, lending, and liquidity in the corporate sector. According to this, increases in collateral values allow greater lending due to the attendant reductions in financial frictions; Greater lending, in turn, increases liquidity in the corporate sector; Finally, increases in corporate liquidity serve to increase collateral values, as these are determined in part by the ability of industry peers to purchase firm assets (Shleifer and Vishny (1992)). Monetary policy affects real outcomes through its impact on this feedback loop between collateral values, lending, and corporate liquidity. By injecting liquidity into the banking sector, unconventional monetary policy shifts banks' lending calculus as they know that increased aggregate lending will influence collateral values.

Our model identifies three mutually exclusive classes of potential equilibria of monetary transmission. In the first equilibrium class, which we call the 'conventional equilibrium', shifts in monetary policy successfully influence aggregate lending activity. This rational expectations equilibrium can be described by the following series of interlocking forces. When the central bank eases monetary policy, the supply of loanable funds increases. Similar to a standard monetary lending channel effect (see e.g. Bernanke and Blinder (1988)), banks will tend to lend out more funds which will increase liquidity in the corporate sector. As liquidity in the corporate sector increases, collateral values rise due to a Shleifer and Vishny (1992) effect: firms become less liquidity constrained, and can hence bid more aggressively when acquiring assets of liquidated firms. As in a standard 'balance sheet channel' effect (e.g. Bernanke and Gertler (1989, 1990, 1995)), the endogenous increase in collateral values improves firms' balance sheets, and thus enables them to borrow the additional liquidity which was injected to the commercial banks by the central bank.

The lending and balance sheet channels of monetary policy are therefore linked in a rational expectations equilibrium through endogenous collateral values: increased bank lending leads to greater liquidity in the corporate sector and thus higher collateral prices. In turn, higher anticipated collateral prices reduce financial frictions and enable banks to utilize the central-bank injection of liquidity to increase lending. In this conventional equilibrium class, an easing of monetary policy thus translates into three effects: an increase in lending, an increase in collateral values, and a

[2]Recent papers which study the interplay between liquidity, fire sales, and asset prices are Acharya and Viswanathan (2009), Acharya, Shin and Yorulmazer (2009), Fostel and Geanakopolos (2008), and Rampini and Viswanathan (2009).

change in the interest rate associated with bank lending.

The second equilibrium class in our model is the 'credit trap' equilibrium. In this equilibrium, any easing of monetary policy beyond a certain point is completely ineffective in increasing lending – banks simply hold on to the additional liquidity created by the central bank. In the credit trap equilibrium aggregate lending is constrained by low collateral values. To increase collateral values the central bank would need to induce banks to inject additional liquidity into the corporate sector so as to increase firms' ability to purchase the assets of other industry participants. However, the marginal increase in collateral values implied by additional lending, and the associated increase in debt capacity, are not sufficiently large to actually induce banks to lend. Regardless of the amount of liquidity added by the central bank, credit therefore remains stuck in banks and collateral values do not increase beyond the low level implied by the lack of corporate liquidity. Note that our notion of a credit trap is different from the traditional liquidity trap in that the former relies on financial frictions and the interplay between liquidity and the value of collateral, whereas the latter emphasizes the inability to enhance real economic activity in a zero interest rate environment.

The third equilibrium class in our model is the 'jump start' equilibrium. In this equilibrium monetary policy can be effective, but only when the central bank acts sufficiently forcefully in injecting liquidity to the banking sector. When increasing capital by only a moderate amount, credit remains trapped in the banking sector as in a credit trap equilibrium. Banks rationally understand that when they can employ only a moderate amount of capital to lend to firms, the implied collateral values are too small to justify any actual lending. Banks, therefore, retain the additional liquidity provided by the central bank as reserves, lending remains low, and in equilibrium the interest rate on loans will remain constant at its lower bound. However, when the central bank uses unconventional monetary policy forcefully, a high lending and high collateral value rational expectations equilibrium arises: lending is high because collateral values are high enough to support it, while collateral values are high because lending increases liquidity in the corporate sector.

The jump start equilibrium class, therefore, provides theoretical support motivated by the credit channel framework for a policy of quantitative or credit easing, showing how, under certain circumstances, such easing can be effective in increasing lending. The jump start equilibrium also explains how small contractions in the stance of monetary policy can lead to large crashes in both asset values and lending. According to this, small reductions in lending reduce liquidity in the corporate sector which, in turn, decreases collateral values. Firms balance sheets are therefore weakened, reducing lending still further. Small reductions in aggregate lending induced by monetary policy are thus amplified, thereby bringing about large contractions in equilibrium lending and collateral values. This effect is very much consistent with accounts of the Japanese experience during the 1980s such as Bernanke and Gertler (1995) who argue that "the crash of Japanese land and equity values in the latter 1980s was the result (at least in part) of monetary tightening; ... [T]his collapse in asset values reduced the creditworthiness of many Japanese corporations and banks, contributing to the ensuing recession."

We model the nature of the equilibrium class that arises – be it a credit trap or jump start – by providing micro foundations for the market for assets. We show that a large aggregate liquidity shock and the resultant asset sales give rise to credit traps in which monetary policy will be ineffective. In contrast, when liquidity shocks are small, the economy will be in a conventional equilibrium and monetary policy will be effective in stimulating lending. When liquidity shocks are at an intermediate level, a quantitative easing policy may be effective in jump-starting lending and investment if monetary policy is pursued sufficiently forcefully. Finally, we show that monetary policy will have limited effectiveness when the level of liquidity in the corporate sector at the time of monetary intervention is low. The model, therefore, shows that monetary intervention may arrive too late: If liquidity in the corporate sector is low, monetary expansions will have limited ability to convey liquidity from financial intermediaries to firms.

As a final point, since the transmission of monetary shocks does not occur through a neoclassical cost-of-capital effect, the model shows how large changes in aggregate lending and investment can be associated with comparatively small changes in interest rates. This result is consistent with empirical evidence showing that monetary shocks have large real effects even though components of aggregate spending are not very sensitive to cost-of-capital variables (see e.g. Romer and Romer (1989), Blinder and Maccini (1991), Bernanke and Blinder (1992), and Christiano, Eichenbaum, and Evans (1996)). The intuition is that an expansion in monetary policy shifts out both loan supply and loan demand – the latter occurring due to the increase in debt capacity associated with the rise in collateral values. Although the outward shift in loan supply and loan demand increase both lending and investment, they have counteracting effects on the equilibrium interest rate. Small changes in interest rates are therefore coupled with large changes in lending and investment.

Our paper belongs in the emerging theoretical literature on the financial crisis of 2008 – 2009. This includes Diamond and Rajan (2009), Kashyap, Rajan and Stein (2008), Shleifer and Vishny

(2010a, 2010b), and Gennaioli, Shleifer and Vishny (2010) which provide a theoretical framework for the crisis based on the role that securitization played in recent years. Bebchuk and Goldstein (2009) offer a slightly different perspective in a model in which credit market freezes arise as a coordination failure amongst banks lending to firms with interdependent projects. Finally, our paper is closely related to Gertler and Karadi (2011) who develop a calibrated model of unconventional monetary policy and evaluate the effectiveness of credit easing during a financial crisis. The main difference between our papers is that Gertler and Karadi (2011) analyze the important question of constrained balance-sheets of financial intermediaries while our paper focuses on collateral constraints in the real sector.

The rest of the paper is organized in the following manner. Section 1 explains the setup of the model. Section 2 analyzes the benchmark case in which liquidation values are determined exogenously. In section 3, which contains the main analysis, we endogenize liquidation values and study their effect on the credit channel transmission of monetary policy. In section 4 we impose more structure on the pricing function of assets by building a micro-founded model of collateral values and the market for assets. Section 5 concludes.

## 1. Model Setup

Consider an economy comprised of a continuous set of self-employed firm-households with measure normalized to unity, a set of commercial banks which can supply capital to firms, and a central bank.[3] Each firm is endowed with a preexisting asset and an identical opportunity to undertake a new project. The project requires an initial outlay of $I$ at date-0, and returns a cash flow of $X_1$ in date-1 and $X_2$ in date-2. As in Hart and Moore (1998) cash flows are assumed to be unverifiable. For simplicity we assume that $I < X_1 < X_2$.[4]

Firms differ in their level of internal wealth, $A$, with $A$ distributed over the support $[0, I]$. For convenience, firms are parameterized by the level of borrowing that they require in order to invest in the project $B = I - A$. We assume that $B$ is distributed according to the cumulative distribution function $G()$, where for simplicity $G$ is twice differentiable with a positive probability density function $g$. Firms can choose between investing in their project and depositing their internal

[3] Our setup of self-employed firm-households is similar to Mendoza (2010).

[4] While by no means necessary, this assumption eases exposition and is consistent with our main interest of tight liquidity in date-1.

wealth with banks, earning a return on deposits at date-1 which will be determined in equilibrium.

We assume that at date-1 a fraction $\gamma$ of households experience a liquidity shock, forcing them to consume their available wealth.[5] As such, $\gamma$ measures the expected severity of the liquidity shock, with high measures of $\gamma$ proxying for large aggregate shocks. When hit by the liquidity shock, firm-households consume the wealth they own, in the process liquidating their asset on the market. Following Shleifer and Vishny (1992), we assume that the only operators of liquidated assets are other firms in the industry – i.e. 'patient' firm-households not hit by the liquidity shock. These firms can use liquidated assets to generate cash flow $V$ at date-2, where for expositional simplicity we assume $V < X_1$. As is common in the fire-sale literature, the market price of liquidated assets, denoted by $L$, may be strictly smaller than $V$. The liquidation value of assets will play a key role in the analysis and will be described further below.

To invest in their project, firms can borrow capital from banks. In so doing, they can pledge their asset as collateral.[6] We further assume that firms cannot issue bonds in the capital markets. While this is a strong assumption, adding a bond market does not change our results qualitatively, as long as banks are assumed to have some informational or monitoring advantage in providing capital.[7] We also assume that owners obtain private benefits of control, $Y$, from running their firms to date-2, the final period of the model, with $Y > V$. This assumption implies simply that firms who are not hit by the liquidity shock do not voluntarily liquidate their assets.[8]

As is common in the literature on the lending channel of monetary policy (see, e.g. Kashyap and Stein, (1994)) the supply of loanable funds is determined by the banks' balance sheet. The right hand side of this balance sheet is comprised of two components: the level of bank capital, $C$, and the level of deposits in the economy, where deposits, as explained above, are generated by firm-households that do not undertake their project. Banks can transform both deposits and their capital into loans. Additionally, banks can hold as reserves in the central bank funds not lent out to firms. For simplicity we assume that the interest rate on reserves is zero as is the reserve requirement on deposits.

Bank capital, $C$, is assumed to be directly determined by the central bank – an institution which

[5] At date-0 the identity of the firms experiencing the shock is unknown.

[6] Note that while the asset is used to collateralize the loan, undertaking the project does not create a new asset.

[7] That intermediated loans are somehow 'special' is a fundamental assumption in the lending channel literature (see Bernanke and Blinder (1988)). Given the information advantage of banks we also assume that firms cannot borrow from or lend to other firms.

[8] An equivalent assumption is to assume that the preexisting asset generates cash flow $X_2' > V$ at date 2. Absent a liquidity shock, firms therefore do not voluntarily liquidate.

is exogenous to the model.[9] Variation in $C$ should be thought of as capturing unconventional monetary policies designed to enhance lending similar to those used by the Federal Reserve during the financial crisis of 2008-2009. These polices included direct lending by the central bank to financial institutions and the purchasing of long term securities from the financial sector. Similarly, equity injections conducted by the Treasury in conjunction with the Federal Reserve directly increased bank capital. The ultimate goal of these unconventional policies was to strengthen bank balance sheets to enhance financial stability and lending. While in the model we refer to monetary policy as influencing bank capital, $C$, this is meant to capture unconventional monetary policy more generally.

While most of our predictions stem from a general equilibrium analysis in which we endogenize the liquidation value of assets, it is useful to begin the analysis with the benchmark case of exogenous liquidation values.

## 2. The Benchmark Case: Exogenous Liquidation Values

We begin by assuming that the liquidation value of the project $L$ is given exogenously. For ease of exposition, we consider the more interesting case where $L < I$.[10]

Consider a firm which needs to borrow an amount $B$ to undertake its project and is faced with an interest rate $r$. Since cash flow is unverifiable, there is no way to induce the firm to repay at date-2. As is common in the literature in incomplete financial contracts, the only method to induce the firm to repay at date-1 is through the threat of liquidation (see, for example, Hart and Moore (1994)). Assuming that at date-1 the firm has all the bargaining power in renegotiating its debt obligation with its bank, the firm will never be able to commit to repay more than $L$ at date-1 as it can always bargain down its repayment to the bank's outside option. Thus, the firm will be able to borrow an amount $B$ only when $B(1+r) \leq L$, or equivalently, when

$$B \leq \frac{L}{1+r}. \tag{1}$$

Rather than undertaking the project, the firm can deposit its funds at a bank, earning net interest $r$ at date-1.[11] Faced with an equilibrium interest rate $r$, a firm will therefore choose to borrow

[9]The sole role of the central bank is to influence $C$ – and so it is not assigned an objective function. In addition, to obtain monetary non-neutrality, we make the standard assumption of imperfect price adjustment.

[10]When $L > I$ the analysis continues to hold but the financial frictions are negligible since liquidation of the asset at the end of the first period yields enough to fully fund investment.

[11]Note that perfect competition between banks will drive up the interest rate on deposits to equal that on loans

$B$ and invest in the project rather than deposit its internal funds in the bank when the following condition holds:

$$\gamma[X_1 + L - (1+r)B] + (1-\gamma)[X_2 + (\frac{X_1 - (1+r)B}{L})V + Y] \geq \gamma[(1+r)(I-B) + L] + (1-\gamma)[(1+r)(I-B)\frac{V}{L} + Y]. \tag{2}$$

Inequality (2) represents the investment participation constraint of firms and reflects also the fact that firms may use date-1 wealth to purchase distressed assets at a price $L \leq V$. The left-hand-side of (2) captures the payoff from investment at date zero. After obtaining $X_1$ from the project, with probability $\gamma$, the firm is hit by the liquidity shock and must consume all available wealth. The firm therefore liquidates its asset, repays the bank $(1+r)B$, and consumes wealth $X_1 + L - (1+r)B$.[12] With probability $1-\gamma$ the firm is not hit by a liquidity shock. The firm uses available cash $X_1 - (1+r)B$ to buy assets at price $L$ in the market. These assets yield a payoff of $V$ in period 2 which jointly with the second-period cash flows $X_2$ and the value of control $Y$ establish the payoff from continuing to the second period. The right-hand-side of (2) provides the payoff for depositing the fund in the bank and is understood in an analogous manner.

Rearranging (2) yields

$$(1-\gamma)X_2 + X_1[\frac{(1-\gamma)V}{L} + \gamma] \geq (1+r)I[\frac{(1-\gamma)V}{L} + \gamma]. \tag{3}$$

As can be seen, (3) is independent of the borrowing requirement $B$. This is intuitive as regardless of whether the firm undertakes the project or deposits funds in the bank, a unit increase in the borrowing requirement, $B$, has the same shadow cost, namely a decrease of $1+r$ in date-1 wealth.

Together, (1) and (3) determine the demand schedule for loans. Defining $\bar{r}$ to be the rate of return for which inequality (3) holds with equality, it is easy to see that for any interest rate $r < \bar{r}$, the participation constraint is non-binding, meaning that all firms would like to borrow. Demand for loans is therefore determined solely by firms' ability to borrow, as given by (1), and is hence equal to:

$$D^*(r) = \int_0^{L/(1+r)} BdG(B). \tag{4}$$

since, by lending out the funds, banks earn $r$ on deposits. The fact that firms will never repay loans using date-2 cash flows also implies that the interest rate on deposits between dates 1 and 2 will be zero, as banks cannot put deposits to productive use between these two periods.

[12] Note that the firm must repay the loan, as the bank can force liquidation and obtain $L$ rather than $(1+r)B$.

At the interest rate $r = \bar{r}$, firms are indifferent between investing and depositing their funds at date-0 in banks. Demand for loans is therefore elastic over the interval $[0, \int_0^{L/(1+\bar{r})}]$.[13] Finally, at any interest rate $r > \bar{r}$, inequality (3) does not hold, implying that demand for loans is zero as no firm would like to borrow.

Figure 1 provides a graphic illustration of loan demand. As can be seen, because of financial frictions – operationalized through the assumption that cash flow is non-verifiable – the demand for loanable funds is determined in part by the *ability* of firms to borrow and not just by their desire to do so. The liquidation value of assets, $L$, thus plays a role in determining the demand for loanable funds through its impact on financial constraints. For $r < \bar{r}$ inequality (1) binds while inequality (3) does not. Demand for loanable funds is determined by firms' ability to borrow, as constrained by liquidation values $L$, rather than their desire to borrow, as determined by the participation constraint. To emphasize this, we refer to the demand function in (4) as 'effective demand', thereby differentiating it from the demand that would have been obtained under no financial frictions.

Turning now to loan supply, the banking sector's loanable funds include bank capital $C$ and deposits by firm-households that do not invest at time 0. For any interest rate $r < \bar{r}$, all firms with borrowing requirement $B \leq L/(1+r)$ undertake the project while those with borrowing requirement $B > L/(1+r)$ deposit their funds in the bank. Recalling that firm internal wealth is given by $A = I - B$, the supply of loanable funds for any interest rate $r < \bar{r}$ is given by

$$S^*(r) = C + \int_{L/(1+r)}^{I} (I - B)dG(B). \tag{5}$$

In contrast, at $r = \bar{r}$ firms are indifferent between investing and depositing their funds, and hence the supply of funds is given by the range $C + \left[ \int_{L/(1+\bar{r})}^{I} (I - B)dG(B), \int_0^I (I - B)dG(B) \right]$. Figure 1 provides a graphic illustration of loan supply.

Equilibrium in the model is determined by equating effective demand for loanable funds to the supply of loanable funds:

$$D^*(r) \leq S^*(r) \ \textit{with strict inequality only when } r = 0. \tag{6}$$

[13] Although at $r = \bar{r}$ firms are indifferent between investing and depositing their funds, we assume for expositional simplicity that, as at all other interest rates, at $r = \bar{r}$ the borrowing set is characterized by an interval $[0, B]$ for some marginal borrowing firm $B$. Our results hold without this assumption. The appendix provides a proof of the key proposition – Proposition 3 – that does not rely on it.

As a matter of terminology, we will say that the market for loanable funds completely clears when (6) holds with equality, so that all funds are lent out and reserves equal zero.

From (4) and (5), it is easy to see that as the central bank increases bank capital, the supply of loanable funds increases. As a result, the equilibrium interest rate will decrease, thereby increasing aggregate lending. Importantly, however, the liquidation value of assets, $L$, will determine the maximal level of aggregate lending. At a zero interest rate, financial frictions imply that the maximal amount any firm can borrow is $B = L$. Thus, for any exogenous $L$, the maximal effective demand is obtained at $r = 0$ and equals $\int_0^L BdG(B)$. If at $r = 0$ loan supply is greater than this maximal loan demand, the market for loanable funds will not clear completely. Aggregate lending will equal its maximal level, $\int_0^L BdG(B)$, with the remainder held by banks as reserves. From (4) and (5) this occurs when:

$$\int_0^L BdG(B) \leq C + \int_L^I (I - B)dG(B) \tag{7}$$

Since internal wealth satisfies $A = I - B$, inequality (7) can be rearranged to yield:

$$IG(L) - E(A) \leq C \tag{8}$$

where $E(A) = \int_0^I (I - B)dG(B)$ is the aggregate date-0 liquidity in the corporate sector under the distribution $G$. Thus, any increase in bank capital, $C$, beyond $IG(L) - E(A)$ will not increase lending to the corporate sector but will instead be held as reserves by banks. In contrast, aggregate lending will be responsive to monetary policy for any $C < IG(L) - E(A)$.[14] Formally, we have:

**Lemma 1.** *For any exogenous liquidation value $L$ and level of bank capital, $C$, define $\hat{r}$ implicitly by*

$$\int_0^{L/(1+\hat{r})} BdG(B) = C + \int_{L/(1+\hat{r})}^I (I - B)dG(B). \tag{9}$$

*The equilibrium is characterized as follows:*

*(i). If $IG(L) - E(A) \leq C$, then the equilibrium interest rate is $r^* = 0$, the equilibrium marginal borrowing firm is $B^* = L$, and aggregate lending is $\int_0^L BdG(B)$.*

*(ii). If $IG(L) - E(A) > C$, then the equilibrium interest rate is $r^* = min(\hat{r}, \bar{r})$, the equilibrium marginal borrowing firm, $B^*$, satisfies $G(B^*) = \frac{1}{I}(C + E(A))$, and aggregate lending is $\int_0^{B^*} BdG(B)$.*

*Proof.* See Appendix.

[14] Notice that lending will not increase one-for-one with increases in bank capital as the reduction in the equilibrium interest rate associated with an increase in $C$ will reduce deposits, partially offsetting the increase in $C$.

To summarize, Lemma 1 shows that the liquidation value of assets limits the effectiveness of monetary policy in inducing lending and investment. Monetary policy itself, however, *shifts* liquidation values through its effect on lending and corporate liquidity. Thus, to understand the actual effectiveness of the transmission mechanism of monetary policy, and in particular to understand when monetary injections of liquidity into the banking sector can induce lending, it is crucial to endogenize the interplay between lending, liquidity and liquidation values.

To do so, for what follows it is useful to define for every amount of loanable funds, $C \leq \int_0^I BdG(B)$, the value $\bar{B}(C)$ which represents the marginal firm that obtains financing assuming that the market for loanable funds clears completely, i.e. all deposits and bank capital are lent out and banks hold no reserves.[15] It is easy to see that $\bar{B}(C)$ is given implicitly by the equation:

$$\int_0^{\bar{B}(C)} BdG(B) = C + \int_{\bar{B}(C)}^{I} (I - B)dG(B). \tag{10}$$

## 3. The Credit Channel with Endogenous Liquidation Values

To endogenize liquidation values, we follow Shleifer and Vishny (1992) and assume that liquidated assets are purchased by other firms within the same industry.[16] Industry participants bid for the defaulted firm's assets, so that demand will be determined both by the potential value of the assets as well as the liquidity constraints of the bidders. As in Shleifer and Vishny (1992), if the liquidity available to the bidders is sufficiently low, the value obtained for the asset will be lower than its first-best value.[17]

Before continuing, it is useful to provide a general description of the model's main effects. The model combines the 'balance-sheet channel' and the 'lending channel' of monetary policy in a general equilibrium rational expectation framework. This can be described with the following series of interlocking forces. When the central bank increases bank capital, the supply of loanable funds increases. Similar to a standard 'lending channel' effect (see e.g. Kashyap and Stein (1995)), banks will tend to lend out more funds, which will increase liquidity in the corporate sector. As liquidity in the corporate sector increases, collateral values rise – firms become less liquidity constrained, and can hence bid more aggressively when acquiring assets of liquidated firms. As in a standard

[15] $C_{max} = \int_0^I BdG(B)$ is the maximal level of aggregate lending possible in the economy.

[16] As is common in these models, banks are assumed to not have the know-how to operate liquidated assets.

[17] Empirical evidence for this industry equilibrium model and its implications for liquidation values, corporate liquidity and debt financing is provided in Benmelech (2009), Benmelech and Bergman (2009) and Pulvino (1998).

'balance sheet channel' effect (e.g. Bernanke and Gertler (1989, 1990, 1995), Lamont (1995)), this endogenous increase in collateral values improves firms' balance sheets, which enhances their ability to borrow the additional liquidity which was injected to the commercial banks by the central bank.

In equilibrium, the lending and balance sheet channels of monetary policy are therefore linked through endogenous liquidation values: increased bank lending leads to greater liquidity in the corporate sector and thus higher collateral prices, while higher collateral prices reduces financial frictions and enables banks to increase lending to firms.

Initially, rather than imposing a particular structure on the market for liquidated assets, we analyze the results using a general specification where the price of assets in liquidation depends on the level of liquidity in the corporate sector and its distribution.[18] Accordingly, we define a pricing function, $P$, for the liquidation value of assets that takes as inputs two variables which jointly span the level and distribution of liquidity at date-1 within the corporate sector. The first variable is $B^*$, the marginal firm that successfully obtained funding at date-0. The second variable is the equilibrium interest rate $r^*$ paid by firms borrowing at date-0. Thus, if a firm is hit by a liquidity shock and liquidates its assets, the price of these assets will be $P = P(B^*, r^*)$, where $P$ is taken to be differentiable in $B^*$ and $r^*$.

We make the reasonable assumption that if date-1 corporate liquidity increases, the price of liquidated assets does not go down.[19] Formally, we assume

**Liquidity Pricing Monotonicity Assumption.**

*(i)* $\partial P/\partial B^* \geq 0$ *for* $r^* = 0$.

*(ii)* $\partial P/\partial r^* \leq 0$ *for any* $r^* > 0$ *and* $B^* \geq \bar{B}(0)$.

These assumptions are straightforward. First, at a zero interest rate, when the proportion of firms obtaining funding at date-0 increases, date-1 liquidity increases, as therefore should the price of liquidated assets.[20] At $r^* = 0$, the pricing function is therefore assumed to be increasing in $B^*$, the marginal firm obtaining finance. Similarly, at any positive interest rate, $r^*$, the market for loanable funds must clear completely. Recalling that for any bank capital level $C$, the marginal firm that obtains financing assuming that the market clears completely is $\bar{B}(C)$, we have that if the interest rate is positive the marginal borrowing firm must satisfy $B^* \geq \bar{B}(0)$. Because date-1

[18] In Section 4 we impose more structure on the pricing function by modeling the market for assets.

[19] Liquidity here refers to the total wealth owned by firms at date-1 which is available to purchase assets.

[20] Throughout the paper, all monotonicity statements refer to weak monotonicity unless stated otherwise.

liquidity is decreasing in the interest rate at which firms borrow, $P$ should therefore be decreasing in $r^*$ whenever $B^* \geq \bar{B}(0)$.

### 3.1. Equilibria with Endogenous Liquidation Values

Given a pricing function $P$, an equilibrium in the lending market is characterized as follows:

**Market Equilibrium.** *An equilibrium in the lending market is a vector $\{C, r^*, L^*, B^*\}$, such that:*
*(i) Firms optimize in their borrowing and investing choices given the interest rate $r^*$ and the liquidation value of assets $L^*$.*
*(ii) Banks optimize in their lending choices, knowing that firms can commit to repay no more than $L^*$.*
*(iii) The market for loanable funds clears at date-0: Denoting by $B^*$ the marginal firm which borrows to invest in a project, the market clearing condition is:*

$$\int_0^{B^*} BdG(B) \leq C + \int_{B^*}^{I} (I - B)dG(B), \text{ with strict inequality only when } r^* = 0$$

*(iv) $L^*$ is an equilibrium liquidation value: $L^* = P(B^*, r^*)$.*

The equilibrium requirements are quite intuitive. First, in equilibrium firms will optimize their borrowing choices. Since each individual firm takes the liquidation value $L^*$ as exogenous, this requirement translates into the optimality conditions developed in inequalities (1) and (3) of the previous section. In optimizing lending decisions, banks will lend at the equilibrium interest rate $r^*$ while understanding that firms cannot commit to repay more than $L^*$. Further, in equilibrium, for any rate $r^* > 0$ realized demand for loanable funds will equal supply. In contrast, when $r^* = 0$ the supply of loanable funds can be greater than the demand – any excess supply will simply be held by the banks as reserves

Finally, equilibrium requirement (iv) is a rational expectations condition, stating that the liquidation value of assets taken as given by individual banks when making their date-0 decisions is indeed the date-1 price of liquidated assets. As described above, this price is determined through a Shleifer-Vishny (1992) equilibrium by the liquidity in the corporate sector and is governed by the pricing function $P$.

We solve for the equilibrium in the following manner. First, the analysis of exogenous liquidation values in Section 2 shows that for every potential liquidation value $L$ and capital level $C$, there exist an associated equilibrium interest rate $r^*$ and equilibrium marginal borrowing firm $B^*$. We can thus define for any liquidation value $L$ and level of bank capital $C$ the associated equilibrium interest rate and marginal borrowing firm, $r^*(L;C)$ and $B^*(L;C)$. Using these, we define for every direct pricing function $P(B,r)$ an *indirect* pricing function:

$$p(L;C) \equiv P(B^*(L;C), r^*(L;C)), \tag{11}$$

which takes as input the liquidation value $L$ and the exogenously given bank capital $C$ and provides as output the implied price of assets given $L$ and $C$.

It is then easy to see that for the rational expectations equilibrium condition (iv) to be satisfied, the equilibrium liquidation value $L^*$ must be a fixed point of $p$ that satisfies $p(L^*;C) = L^*$. If banks at date-0 lend under the assumption that the date-1 liquidation value of assets will be $L^*$, then at date-1, the price of liquidated assets, as determined by the amount of liquidity in the corporate sector in date-1, should indeed be $L^*$. Formally, we have the following proposition:

**Proposition 1.** *Assume an exogenous level of bank capital $C$. Then a market equilibrium $\{C, r^*, L^*, B^*\}$ always exists and $L^*$ is an equilibrium liquidation value if and only if*

$$p(L^*;C) = L^*. \tag{12}$$

*The equilibrium interest rate is then given by $r^*(L^*;C)$, while the marginal firm that borrows in this equilibrium is given by $B^*(L^*;C)$.*

*Proof.* See Appendix.

Using Proposition 1 and employing the solution to the case of exogenous liquidation values yields the following proposition which characterizes the indirect pricing function $p(L;C)$:

**Proposition 2.** *Fix an exogenous liquidation value of assets $L$ and bank capital $C \leq \int_0^I BdG(B)$.*
*(1) For any $L < \bar{B}(C)$:*

*(i) The equilibrium interest rate associated with the pair (L,C) will be $r^* = 0$, and the marginal firm able to borrow will have a borrowing requirement of $B^* = L$.*

*(ii) The indirect pricing function therefore satisfies $p(L;C) = P(L, 0)$.*

*(iii) The market for loanable funds will not clear completely: demand for loanable funds,*

*$\int_0^L BdG(B)$, will be smaller than the supply, $C + \int_L^I (I - B)dG(B)$.*

*(2) For any $L \geq \bar{B}(C)$:*

*(i) The market for loanable funds clears completely, with the entire loan supply lent out.*

*(ii) The marginal borrowing firm has borrowing requirement $\bar{B}(C)$, and the equilibrium interest rate is $r^* = \min\{L/\bar{B}(C) - 1, \bar{r}\}$, with $\bar{r}$ as defined in section 2.*

*(iii) The indirect pricing function satisfies $p(L; C) = P(\bar{B}(C), r^*)$, with $r^*$ given in 2(ii).*

*Proof.* See Appendix.

To understand Proposition 2 consider first a potential equilibrium liquidation value $L$ satisfying $L < \bar{B}(C)$. Since at a zero interest rate the maximal amount firms can borrow is $L$, realized demand at $r = 0$ is $\int_0^L BdG(B)$. Since, by assumption, $L$ is smaller than $\bar{B}(C)$, we have that realized demand at a zero interest rate satisfies:

$$\int_0^L BdG(B) < \int_0^{\bar{B}(C)} BdG(B) = C + \int_{\bar{B}(C)}^I (I - B)dG(B) < C + \int_L^I (I - B)dG(B), \qquad (13)$$

where the equality in (13) is given by the definition of $\bar{B}(C)$, while the second inequality again stems from the assumption that $L < \bar{B}(C)$. Equation (13) shows that, at a zero interest rate, the supply of loanable funds – the right hand side of (13) – is greater than the effective demand for these funds. Because equilibrium interest rates cannot fall further, the equilibrium interest rate associated with any $L$ smaller than $\bar{B}(C)$ will indeed be zero and the associated marginal borrowing firm will have $B = L$. By definition, therefore, the pricing function will satisfy $p(L; C) = P(L, 0)$ on the region $L \leq \bar{B}(C)$.[21] Further, in this region not all of the loan supply will be lent out in equilibrium: realized aggregate lending, $\int_0^L BdG(B)$, will be smaller than loan supply, $C + \int_{\bar{B}(C)}^I (I - B)dG(B)$, and the loan market will not clear completely.

Consider now a potential equilibrium liquidation value $L$ satisfying $L > \bar{B}(C)$. In this case, following the lines of the argument above, we have that at a zero interest rate realized demand, $\int_0^{\bar{B}(C)} BdG(B)$, is greater than the supply of loanable funds, $C + \int_{\bar{B}(C)}^I (I - B)dG(B)$. Thus, in equilibrium, the interest rate will shift upwards so as to equate the supply and demand for loanable funds.[22] Put differently, the equilibrium interest rate will be set such that the marginal firm will have borrowing requirement $\bar{B}(C)$, thereby guaranteeing that $C + \int_{\bar{B}(C)}^I (I - B)dG(B)$ is lent out

[21]Recall that $P$ is the direct pricing function.

[22]In the knife-edge case where $L = \bar{B}(C)$, the equilibrium interest rate will be $r = 0$.

so that the loan market clears completely.

A direct consequence of Proposition 2 which we use in the next section is:

**Corollary 1.** *Fix an exogenous level of bank capital* $C \leq \int_0^I BdG(B)$. *The indirect pricing function* $p(L;C)$ *is increasing in* $L$ *over the region* $L < \bar{B}(C)$ *and decreasing in* $L$ *over the region* $L > \bar{B}(C)$.

Holding $C$ constant, increasing $L$ has two opposing effects on the price of assets in date-1. The first effect is that as $L$ increases, more firms are able to raise external finance which increases liquidity in the corporate sector and therefore raises the market price of date-1 assets. The second effect is that as $L$ increases, more firms are able to borrow. Effective demand for intermediated loans increases, which implies that the equilibrium interest rate of loans rises. An increase in the interest rate reduces liquidity in the corporate sector in date-1, which tends to push down the date-1 price of assets.[23] When $L$ is low the first effect dominates, while when it is high the second dominates. $p(L;C)$ is therefore non-monotonic in $L$.

Combining Proposition 2 with Corollary 1 shows how shifts in monetary policy influence the indirect pricing function. This is illustrated in Figure 2 which presents the impact of an increase in the supply of funds from $C_1$ to $C_2$. By Proposition 2, the pricing function $p(L;C)$ is identical to the function $P(L,0)$ up to the point $\bar{B}(C)$, after which for any $L > \bar{B}(C)$ it is decreasing. As can be seen in the figure, $P(L,0)$ therefore serves as an envelope of $p(L;C)$: for any $C$, the two functions are equal up to the point $\bar{B}(C)$, while $P(L,0)$ is greater than $p(L;C)$ for $L$ greater than $\bar{B}(C)$.

### 3.2. Bank Capital, Liquidation Values, and Lending

In this section we characterize the impact of monetary policy on lending, liquidation values, and interest rates when the value of assets is determined endogenously. We will say that monetary policy is 'ineffective at $C^*$' if, in equilibrium, a marginal increase in bank capital from $C^*$ does not change aggregate bank lending. Conversely, monetary policy is 'effective at $C^*$' if marginal increases in bank capital from $C^*$ strictly increase aggregate lending.

We begin with the following proposition.

[23] This effect is similar to Diamond and Rajan (2001) who show that an adverse effect of liquidity provision is to raise real interest rates which may lead to more bank failures and lower subsequent aggregate liquidity.

**Proposition 3.** *For any level of bank capital $C$ there exists an equilibrium in which the loan market clears completely if and only if $P(\bar{B}(C), 0) \geq \bar{B}(C)$.*

Proposition 3 is quite intuitive. First, for any level of bank capital, $C$, the *minimal* liquidation value of assets required to completely clear the market for loanable funds is $\bar{B}(C)$. To see this note that if the value of assets, $L$, is less than $\bar{B}(C)$, since no firm can borrow more than $L$, the marginal firm able to borrow will have borrowing requirement $B < \bar{B}(C)$. By definition of $\bar{B}(C)$, this then implies that the loan market will not fully clear, with banks holding a positive amount of funds as reserves. Second, for any level of bank capital, $C$, the *maximal* liquidation value of assets when the loan market completely clears is $P(\bar{B}(C), 0)$. This is because, by definition, when the loan market completely clears, the marginal firm obtaining financing will have a borrowing requirement of $\bar{B}(C)$, and liquidity is highest when the interest rate is zero. Proposition 3 therefore states that if the maximal liquidation value of assets conditional on the loan market clearing completely – i.e. $P(\bar{B}(C), 0)$ – is smaller than the minimal liquidation value of assets required to completely clear this market – i.e. $\bar{B}(C)$ – then the loan market will not completely clear. In contrast, the loan market will completely clear if the maximal liquidation value of assets associated with market clearing is greater than the minimal liquidation value of assets required to clear the market. In this case, the equilibrium interest rate and liquidation value will adjust to equate effective loan demand to loan supply, and banks will hold no reserves.

Using Proposition 2 and Proposition 3, we can analyze the general equilibrium effects of shifts in the supply of loanable funds. Proposition 4 provides a formal characterization of three equilibria classes that arise.

**Proposition 4.** *Consider the pricing function $P(L, 0)$.*

(i) **The conventional equilibrium:** If $P(L, 0) > L$ for all $0 \leq L \leq I$ then aggregate investment is strictly increasing in bank capital $C$ over the range $0 \leq C < C_{max}$, where $C_{max} = \int_0^I BdG(B)$ is the maximal level possible of aggregate lending. Monetary policy is therefore effective at any level of bank capital $C < C_{max}$.

(ii) **The credit trap equilibrium:** Assume that $P(L^*, 0) = L^*$, $P(L, 0) > L$ for $0 \leq L < L^*$, and $P(L, 0) < L$ for $L > L^*$. Then monetary policy is effective up to bank capital $C^* = \bar{B}^{-1}(L^*)$ and ineffective beyond $C^*$. Increases in bank capital beyond $C^*$ do not increase lending, nor do they change the equilibrium liquidation value of assets which remains constant at $L^*$. The maximal liquidation value of assets is $L^*$ and maximal aggregate lending is $\int_0^{L^*} BdG(B)$.

(iii) **The jump-start equilibrium:** Assume that $P(L,0) > L$ for $0 \leq L < L_1$, $P(L_i, 0) = L_i$ for i=1,2, and $P(L,0) < L$ over the interval $(L_1, L_2)$. Then, over the region $C \in (\bar{B}^{-1}(L_1), \bar{B}^{-1}(L_2))$ monetary policy is ineffective, the equilibrium liquidation value of assets remains constant at $L_1$, and aggregate lending is constant at $\int_0^{L_1} BdG(B)$. However, at $C = \bar{B}^{-1}(L_2)$, $L_2$ is an equilibrium liquidation value of assets with associated aggregate lending of $\int_0^{L_2} BdG(B)$.

*Proof.* In the Appendix.

To understand Proposition 4 we consider each of the three equilibrium classes in turn.

**Conventional Equilibrium**

First, consider the conventional equilibrium in case (i). Since $P(L,0) > L$ for all $0 \leq L \leq I$, Proposition 3 implies that the market for loanable funds clears completely for any level of bank capital $C$, up to the maximal possible level of lending $C_{max} = \int_0^I BdG(B)$. Monetary policy in this equilibrium is therefore fully effective: increases in $C$ give rise to increases in aggregate lending, up to $C_{max}$.

Figure 3 demonstrates this conventional equilibrium. Increases in $C$ shift out the indirect pricing function $p(L;C)$ as described in Proposition 2. This shift in $p(L;C)$ implies that the equilibrium liquidation value – i.e. the price of assets – will increase (from $L_1^*$ to $L_2^*$ in the figure). The overall chain of events of an increase in loan supply can then be summarized as follows: The increased loan supply is lent out to the corporate sector; the increased liquidity in the corporate sector increases collateral values; and finally, the increase in collateral values increases firm debt capacity, thereby enabling the increase in loan supply.

The effect on equilibrium interest rates of a shift in loan supply is less clear cut. This is demonstrated in Figure 4 which graphs loan supply and loan demand as a function of interest rates, $r$. The main point is that *effective* demand for loans is not just a function of the loan interest rate, but is also influenced by collateral values. Firm borrowing in the model is determined both by their desire to borrow as well as their ability to do so. Loan demand can thus be represented by the function $D(r; L^*)$, where $L^*$ is the equilibrium collateral price.

Figure 4 illustrates two effects of an increase in bank capital, $C$. First, as $C$ increases loan supply shifts out. Second, effective loan demand shifts out as well – when equilibrium liquidation values increase firm borrowing capacity rises. While the outward shifts in loan supply and loan demand both push aggregate lending upwards, they have countervailing effects on the equilibrium interest rate. If loan demand shifts out sufficiently – due to a large increase in collateral prices – the change in the equilibrium interest rate will be small.[24] Put differently, in a conventional equilibrium large changes in aggregate lending and investment can be associated with small changes in interest rates. This is consistent with evidence that monetary shocks have large real effects, even though empirical studies show that components of aggregate spending are not very sensitive to cost-of-capital variables (see e.g. Romer and Romer (1989), Blinder and Maccini (1991), Bernanke and Blinder (1992), and Christiano, Eichenbaum, and Evans (1996)).

**Credit Trap Equilibrium**

Consider now the credit trap equilibrium represented in case (ii) of Proposition 4. In this equilibrium, monetary policy is ineffective at any point beyond $C^*$. The intuition is that for the loan market to clear completely at any level of bank capital beyond $C^*$, liquidation values need to be sufficiently high. However, in a credit trap the implied increase in date-1 liquidation values associated with a marginal increase in bank capital beyond $C^*$ is not sufficient to induce banks to actually lend the additional funds at date-0. Monetary policy thus becomes ineffective above the level of bank capital $C^*$.

The equilibrium is depicted in Figure 5. If the central bank sets bank capital at $C_1 < C^*$, there is a positive equilibrium liquidation value – $L_1$ in the figure – in which the market for loanable funds clears completely. However, monetary policy is completely ineffective at any point beyond $C^*$. As bank capital increases beyond $C^*$, say to $C_2$ in the figure, the sole positive equilibrium liquidation value remains at $L^*$ and equilibrium lending remains constant at $\int_0^{L^*} BdG(B)$. This is because the rate at which the implied value of collateral increases is not sufficiently high to enable banks to lend the additional loan supply. Put differently, banks rationally understand that lending any

[24]In fact, the equilibrium interest rate may actually rise with increases in loan supply. Formally, it is easy to show that the condition for this is $\frac{\partial P(\bar{B}(C), r^*)}{\partial B^*} > 1 + r^*$.That is, the sensitivity of the value of collateral to changes in liquidity (as proxied by $B^*$, the marginal firm obtaining financing) is sufficiently large. We return to this point when discussing jump-start, quantitative easing equilibria.

incremental amount beyond $\int_0^{L^*} BdG(B)$ does not increase collateral values sufficiently to support the additional lending. Since the equilibrium liquidation value $L^*$ does not change with increases in bank capital above $C^*$, firm borrowing capacity remains constant. This implies that realized lending remains at $\int_0^{L^*} BdG(B)$, with leftover loan supply held as reserves by banks. Finally, since beyond bank capital $C^*$ effective loan demand is smaller than loan supply, based on Theorem 2(i), the equilibrium interest rate will remain constant at zero. Monetary policy is powerless in increasing lending, collateral values or corporate liquidity.

To emphasize, note that in this credit trap equilibrium, increased liquidity in the corporate sector would have increased collateral values which could then serve to enable additional lending. The issue, though, is that banks are not willing to supply the additional liquidity on their own. Regardless of the stance of monetary policy, collateral values therefore remain depressed at a low level implied by the lack of liquidity in the corporate sector.

**Jump-start equilibrium**

Consider now the jump-start equilibrium of case (iii) in Proposition 4. As exhibited in Figure 6, for any level of bank capital $C_1 < C < C_2$, the only equilibrium has a liquidation value of $L = L_1$ and associated aggregate lending of $C_1 = \int_0^{L_1} BdG(B))$.[25] Increases in bank capital over the region $[C_1, C_2)$ are therefore completely ineffective in increasing lending and collateral values. There is no response to injections of liquidity by the central bank: the equilibrium liquidation value is stuck at $L_1$ and aggregate lending remains constant at $\int_0^{L_1} BdG(B))$. Further, since as in a credit trap equilibrium, effective loan demand will be depressed due to the low level of liquidation values, in this region the interest rate on loans will be constant at zero, its lower bound.

In contrast, if the central bank acts forcefully enough by increasing bank capital to $C_2$, another equilibrium arises. In this equilibrium the liquidation value of assets is high – $L_2$ in the figure – enabling the market for loanable funds to clear completely with aggregate lending equaling $\int_0^{L_2} BdG(B)$.[26] This equilibrium arises due to the feedback effect between lending and collateral values: lending is high because collateral values are large enough to support it, while collateral values are high because lending increases liquidity in the corporate sector.

The jump-start equilibrium therefore demonstrates how a policy of quantitative easing may

[25]To see this note that for any $C$ in this region, the only value where $p(L; C)$ equals $L$ is $L = L_1$.

[26]Note that by the definition of market clearing, aggregate lending is also equal to loan supply $C_2 + \int_{L_2}^{I} (I - B)dG(B)$.

successfully reignite bank lending. Banks know that when loan supply is moderate, the implied value of collateral conditional on lending occurring is not high enough to actually justify lending. However, when loan supply is expanded sufficiently, a new high lending and high collateral value rational expectations equilibrium arises.

It should be emphasized that although a new equilibrium arises with a sufficiently forceful monetary expansion, it is by no means clear that banks will successfully coordinate on it. If each bank assumes that the others will continue lending at depressed levels, the economy will be stuck in the inefficient equilibrium. In this sense, a policy of quantitative easing in and of itself may not be sufficient to jump-start lending and collateral values. The central bank, or more generally government at large, may require other tools to solve the coordination problem arising between banks.[27]

The jump-start equilibrium also explains how small contractions in the stance of monetary policy can lead to large crashes in asset values and lending. This is simply the flip-side of quantitative easing. Returning to Figure 6, consider an economy with bank capital at $C_2$ and liquidation value at $L_2$. An incremental reduction in bank capital from $C_2$ leads to a collapse in the liquidation value to $L_1$ and a commensurate collapse in lending. The small reduction in lending reduces liquidity and collateral values, which in turn reduces liquidity further. The negative feedback effect between lending, liquidity and collateral values drives all three to lower levels until the process stops at the new equilibrium.

Interestingly, the monetary contraction will also *reduce* the interest rates (to zero according to Proposition 2). The intuition is that while loan supply decreases by a small amount, effective loan demand collapses because of the attendant drop in collateral values. A form of flight to quality arises in which the supply of loans is distributed at low cost to the comparatively small number of firms that have balance sheets strong enough to borrow.

Taken together, therefore, in this equilibrium monetary contraction can lead to crashes in lending and collateral values coupled with a reduction in interest rates. These effects are very much consistent with accounts of the Japanese experience during the 1980s such as Bernanke and Gertler (1995) who argue that "the crash of Japanese land and equity values in the latter 1980s was the result (at least in part) of monetary tightening; ... [T]his collapse in asset values reduced the

[27]To eliminate the low lending equilibrium, these actions may include government subsidies for new loans, a tax on bank reserves, or government prodding to increase lending (as seen, for example, during the crisis of 2008-2009).

creditworthiness of many Japanese corporations and banks, contributing to the ensuing recession."

To conclude, Proposition 4 shows that the efficacy of monetary policy crucially depends on the shape and level of the pricing function $P$ – i.e. on how collateral prices vary with corporate liquidity. What determines the shape of the pricing function – and hence the efficacy of quantitative easing – is therefore a natural question which we turn to in the next section.

## 4. Microfoundations of the Collateral Pricing Function

In this section we impose more structure on the pricing function of assets, $P$, by building a micro-founded model of collateral values and the market for assets. In the model, collateral values are determined in a competitive market for assets. As described in Section 1, at date-1, a fraction $\gamma$ of firms are hit with a liquidity shock and must sell their assets. This supply of assets is absorbed by firms not hit by the shock, who use their date-1 funds to purchase liquidated assets. The equilibrium liquidation value of assets is then determined so as to equate the supply and demand of assets.

As above, we assume that at date-1 all firms – both those that invested at date-0 as well as those that did not – can use their funds to buy liquidated assets. Operating liquidated assets enables firms to generate cash flow $V$ at date-2 with $V < X_1$. For ease of exposition, we further assume that $V > I$, where $I$ is the investment required in the project. As a result, in the first-best outcome in which there are no frictions associated with a lack of liquidity in the corporate sector at date-1, the liquidation value of assets will equal $V$ and all firms will be able to invest in the project at date-0.

To obtain the pricing function $P(B, r)$, we calculate aggregate date-1 liquidity assuming an interest rate of $r$ and a marginal borrowing firm with borrowing requirement $B$. This is given by:

$$Q(B, r) = (1 - \gamma)\Big[\int_0^{L/(1+r)} (X_1 - B(1 + r))dG(B) + \int_{L/(1+r)}^{I} (I - B)(1 + r)dG(B)\Big], \qquad (14)$$

where the first integral in the square brackets reflects aggregate wealth of firms that invested in the project in date-0, the second integral reflects aggregate wealth of those firms that deposited their funds in a bank and the $(1 - \gamma)$ factor represents the fraction of firms able to purchase assets at date-1.

Given aggregate date-1 liquidity $Q(B, r)$, the demand schedule for assets is given in the following manner. At any price $0 < P < V$, the demand for assets is simply $\frac{Q(B,r)}{P}$. At the price $P = V$, firms are indifferent to purchasing liquidated assets, and hence demand is completely elastic up to the maximal level $\frac{Q(B,r)}{V}$.

Given aggregate liquidity $Q(B,r)$, the demand for assets is therefore:

$$D(P;B,r)=\begin{cases}[0,\frac{Q(B,r)}{V}] & if\ P=V\\ \frac{Q(B,r)}{P} & if\ P\in(0,V)\end{cases} \tag{15}$$

and the market clearing condition is $D(P;B,r)=\gamma$. As a result, the equilibrium price of assets, $P(B,r)$, is given by:

**Lemma 2.** *Given an interest rate $r$ and a marginal borrowing firm $B$, the market clearing price of assets at date-1 is given by:*

$$P(B,r)=\min\{\frac{Q(B,r)}{\gamma},V\}. \tag{16}$$

*Proof. See Appendix.*

Lemma 2 simply incorporates the fact that the market clearing price of liquidated assets must satisfy $\frac{Q(B,r)}{P}=\gamma$, so long as this price does not exceed $V$, as firms are not willing to pay more than $V$ for liquidated assets.

The following corollary verifies that the pricing function $P(B,r)$ satisfies the monotonicity assumptions (i) and (ii) of Section 3, enabling us to build on the analysis provided in that section.

**Lemma 3.** *$P(B,r)$ given in equation (16) satisfies the monotonicity assumptions (i) and (ii) of Section 3.*

*Proof.* See Appendix.

Given that the required assumptions of Section 3 are satisfied, Proposition 4 holds. This implies that the equilibrium regime that arises depends crucially on the behavior of the pricing function $P(L,0)$. From (16) we have that $P(L,0)=\min\{\frac{Q(L,0)}{\gamma},V\}$, where $Q(L,0)$ is aggregate date-1 liquidity available to purchase assets assuming a zero interest rate and a marginal borrowing firm $B=L$. As firm wealth satisfies $A=I-B$, we rearrange (14) to obtain:

$$\begin{aligned} Q(L,0) &= (1-\gamma)\Big[\int_0^L (X_1-B)dG(B)+\int_L^I (I-B)dG(B)\Big], \\ &= (1-\gamma)\Big[(X_1-I)G(L)+E(A)\Big] \end{aligned} \tag{17}$$

where $E(A)=\int_0^I(I-B)dG(B)$ is aggregate date-0 wealth. The first term in the square brackets of equation (17) is simply net aggregate wealth created from date-0 investment in projects: $G(L)$

is the fraction of the population obtaining financing at date-0, while $X_1 - I$ is the per-project net addition in date-1 wealth. Aggregate date-1 liquidity is then simply aggregate date-0 wealth, $E(A)$, combined with aggregate wealth created from date-0 investment, multiplied by $(1-\gamma)$ which reflects the liquidity shock.

Using equations (16) and (17), we have:

$$P(L,0) = \min\{\frac{(1-\gamma)}{\gamma}[(X_1 - I)G(L) + E(A)], V\}. \tag{18}$$

Ignoring the minimization in (18), the pricing function $P(L,0)$ is an affine transformation of the distribution of firm borrowing requirements, $G(L)$. As such, the shape of the pricing function is governed by $G(L)$, and in particular is increasing in $L$. When more firms obtain financing, date-1 liquidity available to purchase assets increases which naturally increases the equilibrium price of assets. The price of assets increases in $L$ until date-1 firm aggregate liquidity is sufficient to purchase the entire supply of liquidated assets, $\gamma$, at their full value $V$.

The following two propositions analyzes the effect of the magnitude of the liquidity shock, $\gamma$, on the equilibrium outcome.

**Proposition 5.** *There exists a* $\underline{\gamma} > 0$ *such that for all* $\gamma \leq \underline{\gamma}$*, the equilibrium liquidation value satisfies* $L^* = V$ *and the economy will be in a conventional equilibrium.*

*Proof.* See Appendix.

When the probability of a liquidity shock, $\gamma$, is low, date-1 corporate liquidity is comparatively high relative to the amount of assets that are being liquidated on the market. Lack of liquidity in date-1 will therefore not be a constraint, and the market price of assets will equal its full value $V$. Put differently, liquidation values are robust to liquidity shocks that are not systemic in that only a comparatively small fraction of assets are liquidated. Regardless of the level of date-0 capital injections by the central bank, aggregate wealth in the corporate sector is sufficient to absorb the asset supply being liquidated on the market without having the price of assets drop below $V$. Finally, because $V > I$, sufficient injections of liquidity by the central bank into the banking sector will enable all firms to borrow and invest.

In contrast to the case where the liquidity shock is comparatively small, the next proposition analyzes the effect of a large liquidity shock on the liquidation value of assets and the efficacy of monetary policy.

**Proposition 6.** *For any $\gamma$, define $\bar{L} = \max_C\{L^*\}$ to be the maximum equilibrium liquidation value of assets given any stance of monetary policy. Then, there exists $\overline{\gamma} > 0$ such that for any $\gamma > \overline{\gamma}$:*

(i) The maximal equilibrium liquidation value $\bar{L}$ is strictly less than $V$, and monetary policy is ineffective beyond $\bar{B}^{-1}(\bar{L})$.

(ii) $\bar{L}$ is strictly decreasing in the intensity of the liquidity shock, $\gamma$.

(iii) As $\gamma \to 1$ we have that $\bar{L} \to 0$.

*Proof.* See Appendix.

Proposition 6 shows how a large liquidity shock reduces the liquidation value of assets and inhibits the effectiveness of monetary policy. When a large fraction of firms are hit with a liquidity shock – i.e. $\gamma$ is sufficiently large – liquidation values decline as large quantities of assets need to be purchased by a corporate sector whose aggregate liquidity is low. Liquidity pricing is in effect with liquidation values dropping to below their full value $V$. Knowing that liquidation values will be low, banks curtail lending in the face of severe liquidity shocks. As in a credit trap, while date-0 lending would have served to increase date-1 aggregate liquidity and with it the price of collateral, the implied value of collateral is too low to actually justify the lending. The economy therefore suffers from a lack of liquidity, collateral prices are depressed, and lending remains low regardless of the stance of monetary policy.

To emphasize, the collateral pricing function $P(L, 0)$ *does* increase beyond the maximal liquidation value $\bar{L}$. However, to enable the value of collateral to increase beyond this level, banks need to supply liquidity to firms at date-0 to increase date-1 liquidity available to purchase liquidated assets. This, however, will not occur. Regardless of the stance of monetary policy, banks will not lend any level of loan supply greater than $\bar{B}^{-1}(\bar{L})$ since the implied value of collateral is not sufficiently high to enable the lending. Expectations of a systemic liquidity shock create, therefore, a form of economy-wide 'asset overhang': to push up asset prices, banks need to increase lending sufficiently forcefully. Because of expected asset sales, however, the level of lending required to generate an increase in the value of collateral above $\bar{L}$ is larger than what can be supported by the resultant value of collateral. In equilibrium, expectations of asset sales and low future corporate liquidity depress lending and, further, make monetary policy ineffective beyond a certain threshold. As a final point, increases in the severity of the liquidity shock, $\gamma$, decrease corporate liquidity while increasing the supply of assets being liquidated on the market. Hence, as Proposition 6 states, when the severity of liquidity shocks intensifies, the price of collateral declines and the efficacy of monetary policy is reduced.

The following two propositions show that for intermediate levels of the liquidity shock $\gamma$ a jump-start equilibrium can arise. Specifically, Proposition 7 shows that for intermediate $\gamma$, a policy of quantitative easing will be effective when aggregate date-0 wealth is small and the density of firms with high internal wealth is low.

**Proposition 7.** *For any $\delta$ arbitrarily small there exist $A_0$ and $\epsilon$ with the following property. For any distribution $G$ with date-0 aggregate wealth $E(A) < A_0$ and $\max_{L \in [0,\delta]} g(L) < \epsilon$, there exist a $\gamma_1$ and $\gamma_2$ such that for any $\gamma \in [\gamma_1, \gamma_2]$, a policy of quantitative easing will be successful in increasing lending. Specifically, for any $\gamma \in [\gamma_1, \gamma_2]$ there exists a $\underline{C}$ with:*
*(i) Monetary policy ineffective over the region $[0, \underline{C})$.*
*(ii) For $C \in [0, \underline{C})$, the equilibrium value of assets, $L_0$, is constant and independent of loan supply, and lending is constant at $\int_0^{L_0} BdG(B)$.*
*(iii) If bank capital is increased to $C = \underline{C}$, a new equilibrium arises with collateral value $L_1 > L_0$ and lending equal to $\int_0^{L_1} BdG(B)$.*

*Proof.* See Appendix.

Proposition 7 shows that when aggregate date-0 wealth is low and there are few firms with high internal wealth a policy of quantitative easing will be effective. While moderate levels of loan supply will not be lent out, after sufficiently large injections of liquidity, a high-lending equilibrium arises.

The intuition is as follows. When there are few firms with high internal wealth, in order for banks to be willing to lend out moderate levels of loans, expected date-1 collateral values must be comparatively high. This is because if the value of collateral is low, only the relatively few high internal wealth firms will be able to borrow, which would imply low levels of lending. However, because initial date-0 liquidity is low – i.e. E(A) is low – moderate levels of lending and investment do not create enough date-1 liquidity to increase collateral values to a level sufficiently high to extract the moderate level of lending. Moderate levels of lending, therefore, do not occur in equilibrium. Only when lending is sufficiently high, corporate investment and date-1 liquidity increase to the point where collateral values are high enough to enable lending. Thus, a jump-start, quantitative easing equilibrium arises in which sufficiently forceful liquidity injections give rise to a high collateral and high lending equilibrium. Note that the constraint on the size of the liquidity shock $\gamma \in [\gamma_1, \gamma_2]$ guarantees that $\gamma$ is not so high to imply a credit trap as in Proposition 6, nor is it so low so as to guarantee a conventional equilibrium as in Proposition 5.

We now turn to analyze the effect of underlying productivity on the likelihood that a quantitative easing policy will be successful in increasing bank lending and collateral values. We show that when $\gamma$ is at an intermediate level, quantitative easing will be successful when there are fewer firms with relatively high internal wealth but the productivity of investment is sufficiently high.

**Proposition 8.** *Assume that the distribution of firm borrowing requirements, $G$, is strictly convex. If date-1 project output, $X_1$, is sufficiently high, there exist $\gamma_1$, $\gamma_2$ such that a policy of quantitative easing will be successful in increasing lending for any $\gamma \in [\gamma_1, \gamma_2]$.*

*Proof.* See Appendix.

The intuition for Proposition 8 is as follows. A convex distribution of borrowing requirements, $G$, implies that the distribution of firms is skewed towards those with less wealth. As a result, initially, increases in collateral values do not enable large increases in investment since only high internal wealth firms can borrow. Therefore, date-1 liquidity is initially relatively insensitive to increases in collateral values. As a result, moderate levels of collateral values cannot occur in equilibrium since the level of investment they enable does not create enough date-1 liquidity to increase expected collateral values sufficiently. Because moderate levels of collateral values cannot arise in equilibrium, moderate levels of lending will not be lent out: banks realize that collateral values will be too low to support such lending. However, if investment is sufficiently productive – i.e. $X_1$ is sufficiently high – high levels of lending and investment can occur in equilibrium as banks understand that the resultant date-1 liquidity will give rise to high collateral values. Sufficiently forceful injections of liquidity into the banking sector can thus create a high lending - high collateral value equilibrium.

### 4.1. The Effects of Initial Liquidity

Finally, we analyze how E(A), the date-0 level of liquidity in the corporate sector, affects the success of monetary policy interventions. The following proposition shows that if investment is not sufficiently productive, in that $X_1$ is sufficiently small, low levels of date-0 aggregate corporate liquidity necessarily involve credit traps:

**Proposition 9.** *Assume that* $X_1 < \frac{I}{1-\gamma}$. *Then there exists a threshold level of aggregate liquidity* $\bar{A}$ *and a level of bank capital* $C^* < \int_0^I BdG(B)$ *such that for any distribution of date-0 liquidity* $G$ *with aggregate liquidity* $E(A) < \bar{A}$, *monetary policy will be ineffective beyond* $C^*$.

*Proof.* See Appendix.

The intuition of Proposition 9 is as follows. In providing liquidity to the corporate sector, banks must rely on the aggregate liquidity *already present* in the corporate sector, as this determines, in part, the value of collateral and the strength of firms' balance sheets. Therefore, when injecting liquidity into the banking sector in the hope of increasing bank lending and jump starting the feedback loop between lending and collateral, the central bank must also rely on the initial level of liquidity in the corporate sector. In effect, the central bank is leveraging the initial level of aggregate liquidity to inject additional liquidity into the corporate sector. Proposition 9 then states that if this initial level of aggregate liquidity is sufficiently low, the central bank's ability to leverage existing liquidity to increase lending will be limited – i.e. the economy will be in a credit trap equilibrium where banks will not increase lending beyond a certain level, regardless of shifts in the loan supply.

The next proposition states that if an economy is in a credit trap, decreases in date-0 corporate liquidity intensify the severity of the credit trap: the maximal level of lending and the maximal value of collateral both decrease.

**Proposition 10.** *Consider an economy with a date-0 distribution of borrowing needs* $G_1$ *which is in a credit trap equilibrium with a maximal liquidation value of assets* $\bar{L}_1$. *If* $G_2$ *first order stochastically dominates* $G_1$, *implying that date-0 liquidity in the corporate sector is higher under* $G_1$ *than under* $G_2$, *we have that:*

*(i) Under* $G_2$, *the maximal liquidation value of assets,* $\bar{L}_2$, *will be smaller than* $\bar{L}_1$.

*(ii) Maximal aggregate lending under* $G_2$ *will be smaller than under* $G_1$.

*Proof.* See Appendix.

Proposition 9 and 10 make clear the importance of initial aggregate liquidity when the central bank tries to intervene and inject liquidity into the banking sector. Proposition 9 states that if aggregate liquidity is sufficiently low at the point of intervention, the economy will be stuck in a credit trap, while Proposition 10 states that as the aggregate liquidity decreases, this credit trap becomes more severe. Put together, the propositions show that monetary intervention may arrive too late: If liquidity is sufficiently low in the corporate sector, monetary expansions will not easily convey additional liquidity from financial intermediaries to firms.

## 5. Conclusion

We study the limitations of the unconventional monetary policy in stimulating lending. Using a model that relies on the interplay between lending, liquidity, and collateral values, we identify three equilibrium classes that may arise. The first equilibrium class is one in which monetary policy successfully influences aggregate lending activity. In this conventional equilibrium, liquidity injections into banks translates into an increase in collateral values and lending. In the second equilibrium class, which we call a credit trap equilibrium, the transmission mechanism of monetary policy fails. Any easing of monetary policy beyond a certain level is completely ineffective in increasing aggregate lending or collateral values. In the third equilibrium class, called the jump-start equilibrium, a policy of quantitative easing will be successful in increasing bank lending: monetary policy can be effective, but only when the central bank injects a sufficiently large amount of capital into the banking sector.

We have shown how financial frictions and the interplay between liquidity and collateral values hinder the translation of liquidity injections to the financial sector into increased credit and investment. This line of reasoning suggests that *direct* injections of liquidity into the corporate sector may be beneficial. In particular, by circumventing financial intermediaries, liquidity provision to the corporate sector will increase collateral values directly, enabling firms to extract liquidity from banks on their own. As such, fiscal policy may play an important role in boosting lending and investment. We leave this topic for future research.

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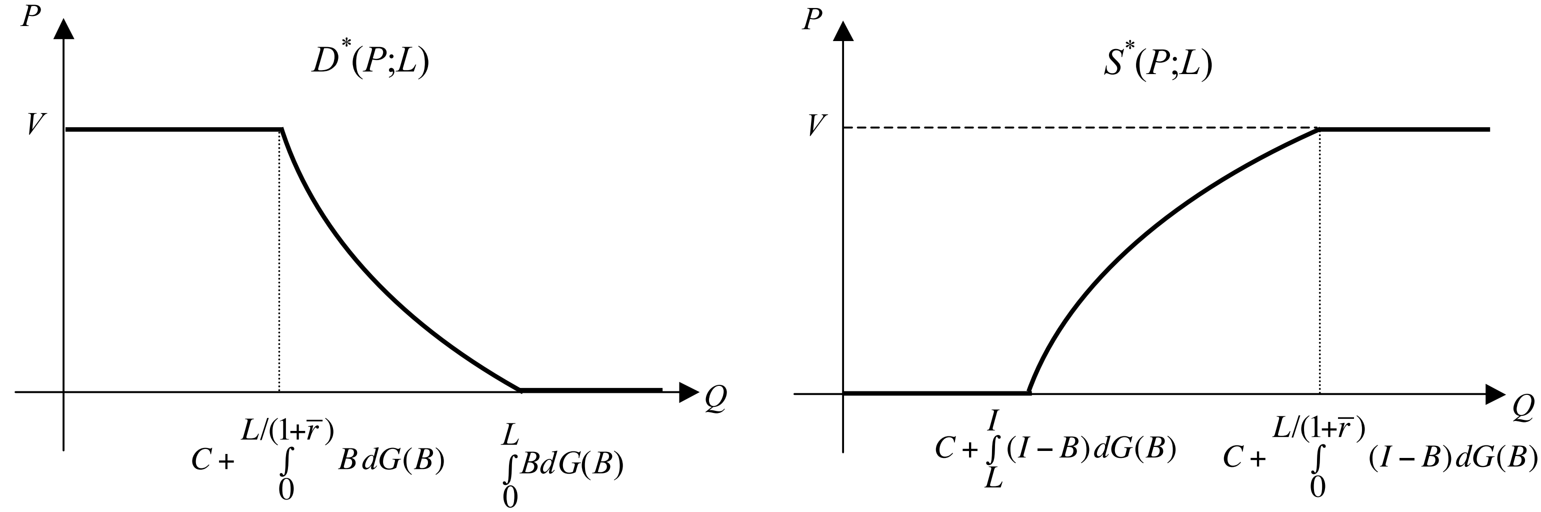


**Figure 1.** Demand for loanable funds (left) and supply of loanable funds (right) for the case of an exogenous liquidation value, $L$, and bank capital level, $C$.

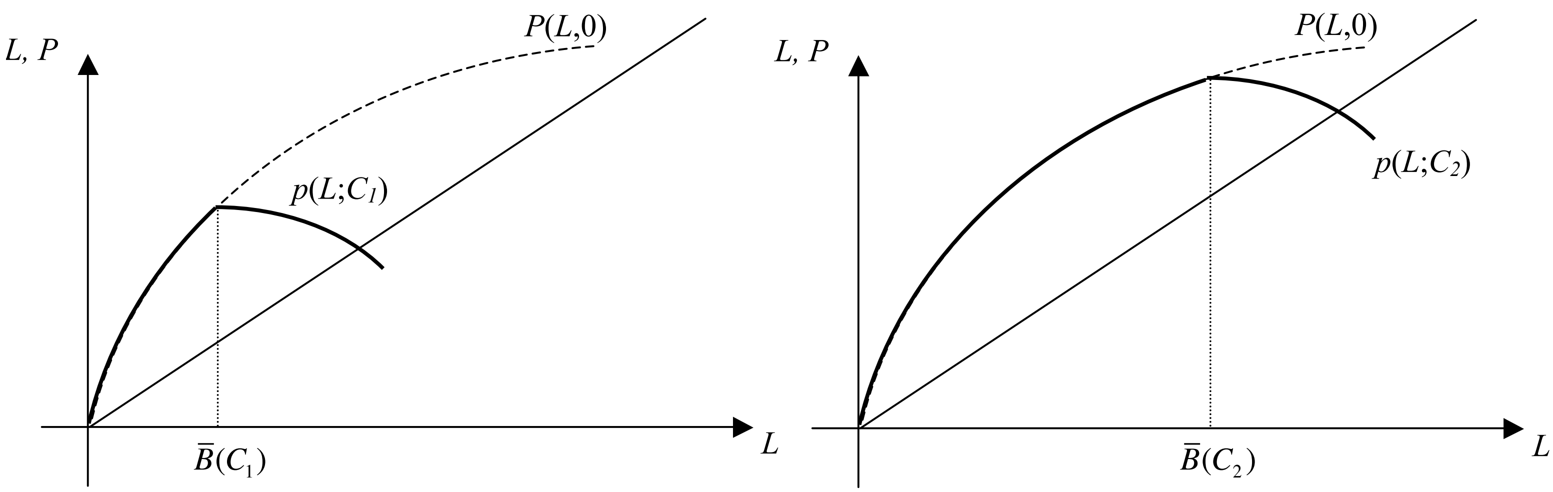


**Figure 2.** The indirect pricing function $p(L;C)$ provides the implied price of assets assuming a liquidation value $L$ and bank capital $C$. As $C$ expands, the direct pricing function $P(L,0)$ serves as an envelope of $p(L;C)$.

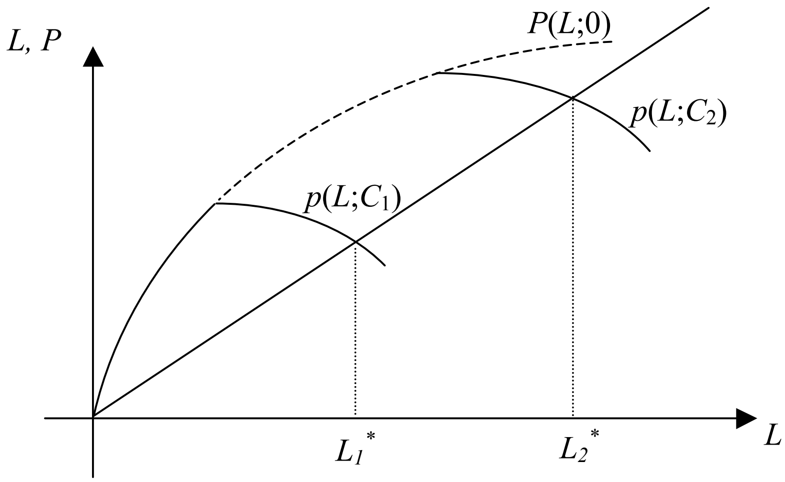


**Figure 3.** The conventional equilibrium. This figure presents the pricing function $p(L;C)$ for two levels of bank capital, $C_1$ and $C_2$. The equilibrium liquidation value increases from $L_1^*$ to $L_2^*$.

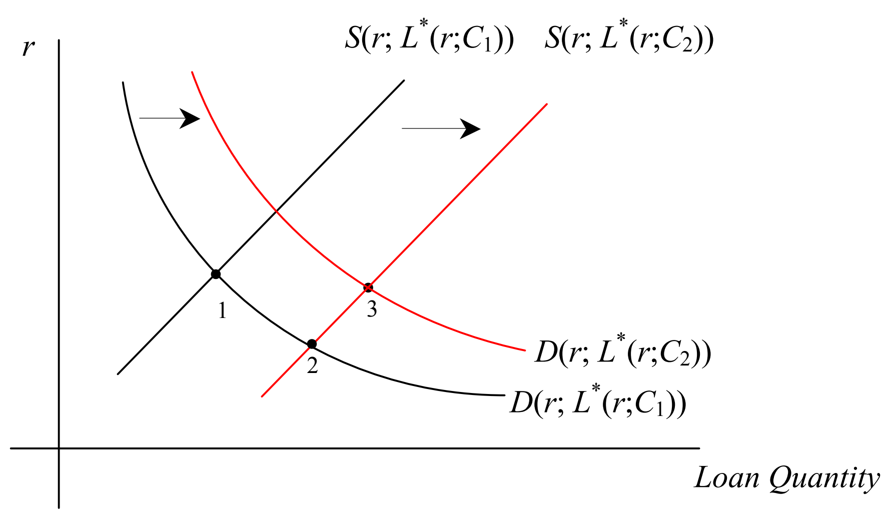


**Figure 4.** The market for loanable funds. Aggregate loan supply and aggregate loan demand as a function of interest rate $r$ for two levels of bank capital, $C_1$ and $C_2$. An increase in $C$ shifts out both loan supply and effective loan demand.

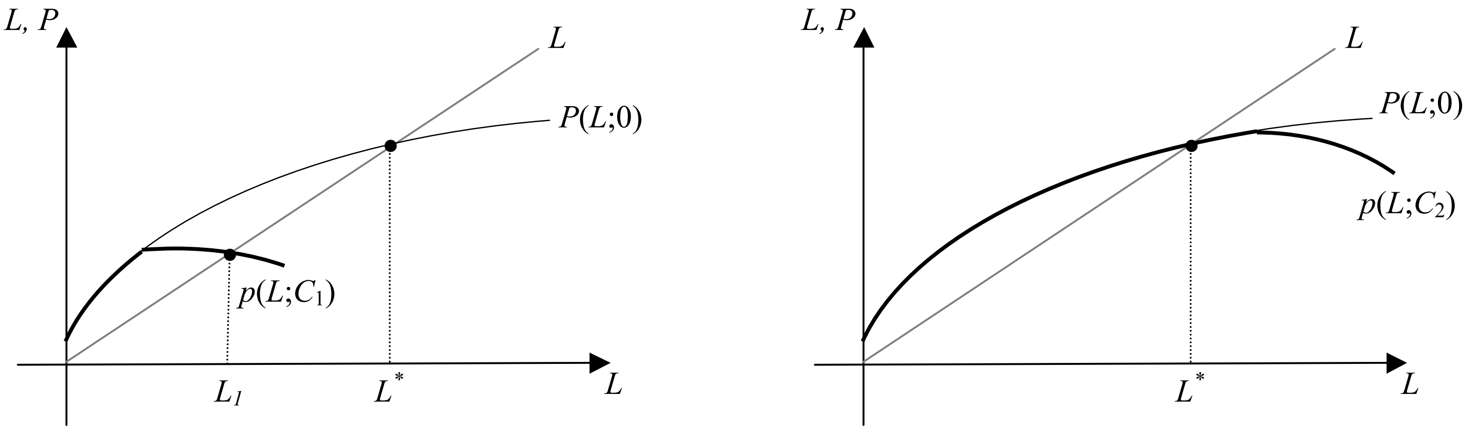


**Figure 5.** The credit trap equilibrium. The pricing function $p(L;C)$ represents the implied price of assets assuming a liquidation value $L$ and bank capital $C$. Increases in bank capital beyond $C^* = \bar{B}^{-1}(L^*)$ do not increase the equilibrium value of collateral or equilibrium lending.

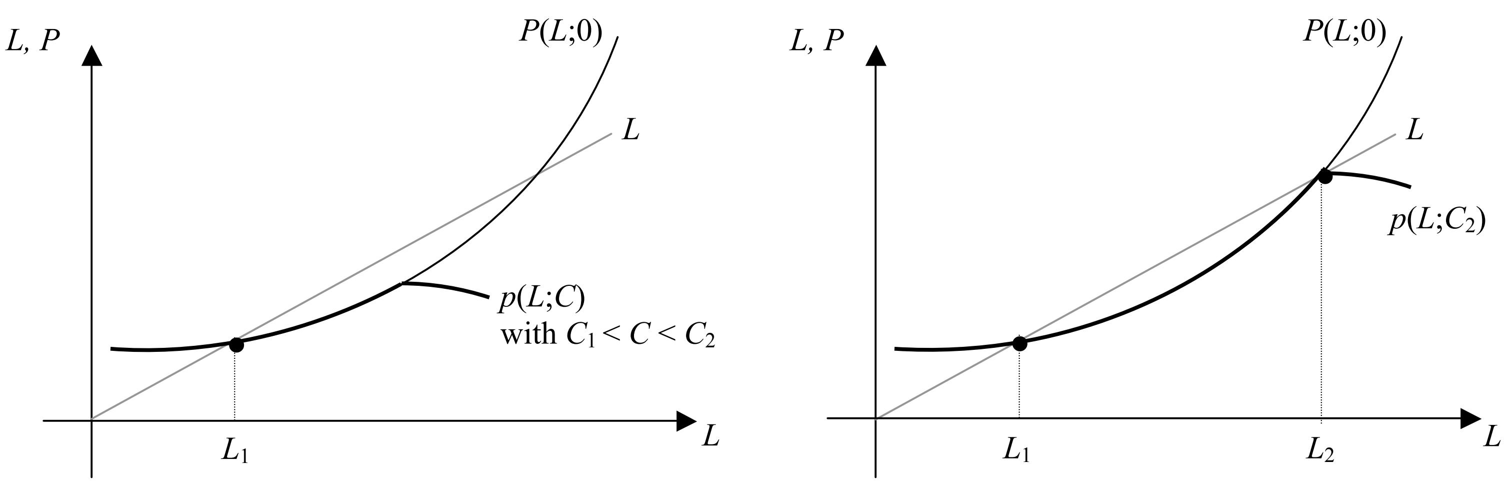


**Figure 6.** The jump-start equilibrium. The pricing function $p(L;C)$ represents the implied price of assets assuming a liquidation value $L$ and bank capital $C$. Monetary policy is ineffective over the region $(C_1, C_2)$, where $\bar{B}(C_i) = L_i$, $i = 1,2$.

## Appendix (Not for Publication)

**Proof of Lemma 1.** (i) The equilibrium borrowing firm must have $B^* \le L/(1+r^*)$, since otherwise it cannot borrow. Suppose $r^* > 0$. Then the loan market clears, so that $\int_0^{B^*} BdG(B) = C + \int_{B^*}^I (I - B)dG(B)$, which implies that $IG(B^*) - E[A] = C$. However, when $r^* > 0$, $B^* \le L/(1+r^*) < L$, so $IG(L) > IG(B^*) = E[A] + C$, a contradiction. Therefore, $r^* = 0$. Since $X_1 > I$, every firm would like to borrow, and thus firms who are borrowing in equilibrium are those who can borrow. Hence, $B^* = L$, and the aggregate lending is $\int_0^L BdG(B)$.
(ii) The proof of (i) implies that in an equilibrium where $r^* = 0$, $B^* = L$. For this to be an equilibrium, we must have $\int_0^L BdG(B) \le C + \int_L^I (I - B)dG(B)$, which implies that $IG(L) - E[A] \le C$, a contradiction. Therefore, $r^* > 0$. The loan market clears when $r^* > 0$, so $\int_0^{B^*} BdG(B) = C + \int_{B^*}^I (I - B)dG(B)$, which implies that $G(B^*) = (C + E[A])/I$ and that the aggregate lending is $\int_0^{B^*} BdG(B)$. Now $\hat{r}$ is defined so that $IG(L/(1+\hat{r})) = E[A] + C$, so $B^* = L/(1+\hat{r})$. Since in equilibrium $B^* \le L/(1+r^*)$, we have $r^* \le \hat{r}$. If $r^* > \bar{r}$, then all firms strictly prefer not borrowing, and the loan market cannot clear. Therefore, $r^* \le \min\{\hat{r}, \bar{r}\}$. It remains to show that $r^* \ge \min\{\hat{r}, \bar{r}\}$. Suppose that $r^* < \min\{\hat{r}, \bar{r}\}$. Then all firms strictly prefer borrowing, and thus $B^* = L/(1+r^*)$, which implies that $r^* = \hat{r}$, a contradiction.

**Proof of Proposition 1.** First assume that $L^*$ satisfies $p(L^*; C) = L^*$. By definition, $B^*(L; C)$ and $r^*(L; C)$ are the associated equilibrium marginal borrowing firm and market clearing interest rate associated with the exogenous pair of liquidation values and bank capital $(L, C)$. By construction, therefore, the vector $(C, r^*, L^*, B^*)$ satisfies conditions (i) through (iii) of a market equilibrium. Further, we have that $L^* = p(L^*; C) = P(B^*(L^*; C), r^*(L^*; C))$, where the second equality results from the definition of the pricing function $p$. Thus, condition (iv) of the market equilibrium is satisfied as well, guaranteeing that $C, r^*, L^*, B^*$ is indeed an equilibrium.

Suppose now that the vector $(C, r^*, L^*, B^*)$ is a market equilibrium. By Section 3, it is easy to see that for every exogenous pair $(L^*, B^*)$ there is a unique equilibrium marginal borrowing firm and market clearing interest rate. Thus, $B^*(L^*; C) = B^*$ and $r^*(L^*; C) = r^*$. Since $(C, r^*, L^*, B^*)$ is a market equilibrium, condition (iv) implies that $L^* = P(B^*, r^*) = P(B^*(L^*; C), r^*(L^*; C)) = p(L^*; C)$. $L^*$ is therefore a fixed point of $p$ as required.

As a final point, note that existence of at least one equilibrium is guaranteed by the fact that $p$ is continuous (since $P$ is continuous) and bounded from above by $X_1$.

**Proof of Proposition 2.** Suppose that $r^* > 0$ in equilibrium. Let $B^*$ be the marginal borrower in equilibrium. Then $B^* \le L/(1+r^*)$ as only firms with borrowing requirement not exceeding $L/(1+r^*)$ are able to borrow. Then the clearing of the loan market implies that

$$\int_0^{B^*} BdG(B) = C + \int_{B^*}^I (I - B)dG(B),$$

or equivalently,

$$C = \int_0^{B^*} BdG(B) - \int_{B^*}^I (I - B)dG(B).$$

The right hand side is strictly increasing in $B^*$, and $B^* \leq L/(1+r^*) < \bar{B}(C)$, so the right hand side is less than $\int_0^{\bar{B}(C)} BdG(B) - \int_{\bar{B}(C)}^{I}(I-B)dG(B)$, which equals $C$ by definition of $\bar{B}(C)$, a contradiction. Therefore, the equilibrium interest rate must be zero.

Note that it is assumed that $X_1 > I$, so when the equilibrium interest rate is zero, every firm *would like* to borrow and invest. Therefore, all the firms with borrowing requirement $B \leq L$ are borrowing, while those with $B > L$ are not *able* to borrow. Therefore, at a zero interest rate the marginal borrowing firm will have a borrowing requirement of $L$; in other words, $B^* = L$. It is easy to check that this is indeed a market equilibrium. By definition, the indirect pricing function is therefore $p(L;C) = P(L,0)$. Finally, in this equilibrium, the level of loan supply actually lent out will equal the effective loan demand at the zero interest rate, $\int_0^L BdG(B)$. Since $L < \bar{B}(C)$,

$$C = \int_0^{\bar{B}(C)} BdG(B) - \int_{\bar{B}(C)}^{I}(I-B)dG(B) > \int_0^L BdG(B) + \int_L^I (I-B)dG(B),$$

which implies that the demand for loanable funds is smaller than the total loan supply.

When $L = \bar{B}(C)$, the above argument implies that a) there cannot be an equilibrium with $r^* > 0$, and b) $(r^*, B^*) = (0, L)$ is a market equilibrium, which obviously satisfies 2(i), (ii), and (iii).
Consider now the case where $L > \bar{B}(C)$. By the above argument, in an equilibrium with zero interest rate, the marginal borrower is $L$ and $\int_0^L BdG(B) \leq C + \int_L^I (I-B)dG(B)$, which cannot be true as $L > \bar{B}(C)$. Therefore, the equilibrium interest rate must be positive. It follows that the entire loan supply is lent out, and therefore the market clearing condition $\int_0^{B^*} \tilde{B}dG(\tilde{B}) = C + \int_{B^*}^{I}(I-\tilde{B})dG(\tilde{B})$ implies that $B^* = \bar{B}(C)$. Since the marginal borrower must be able to borrow, $\bar{B}(C) \leq L/(1+r^*)$, or $r^* \leq L/\bar{B}(C) - 1$. On the other hand, the marginal borrow has to be willing to borrow, so $r^* \leq \bar{r}(L)$. Note that in all the proofs we will write $\bar{r}(L)$ to emphasize the dependence of $\bar{r}$ on $L$.

If the marginal borrower is indifferent between borrowing and not borrowing, then the analysis that proceeds the proposition implies that $r^* = \bar{r}(L)$. Otherwise, since a firm's participation constraint is independent of its borrowing requirement, all firms strictly prefer borrowing to not borrowing. Therefore, it must be the case that in equilibrium only firms with borrowing requirement at most $\bar{B}(C)$ can borrow, which implies that $\bar{B}(C) = L/(1+r^*)$, or $r^* = L/\bar{B}(C) - 1$. It is easy to check that we have obtained a market equilibrium in both cases. Finally, since the equilibrium marginal borrowing firm and interest rate are $\bar{B}(C)$ and $r^*$ respectively, we have that the indirect pricing function satisfies $p(L;C) = P(\bar{B}(C), r^*)$.

**Proof of Corollary 1.** Consider first a liquidation value $L$ with $L \leq \bar{B}(C)$. By Proposition 2, we have that in this region $p(L;C) = P(L,0)$. Since $P(L,0)$ is increasing in $L$, $p$ will be increasing in $L$ as well in this region (recall that, unless stated otherwise, throughout the paper 'increasing' refers to weak monotonicity.)

Similarly, by Proposition 2, if $L > \bar{B}(C)$ we have that

$$p(L;C) = P(\bar{B}(C), r^*), \tag{1}$$

with $r^* = \min\{\bar{r}(L), L/\bar{B}(C) - 1\}$ by Proposition 2. It is easy to see that $\bar{r}(L)$ is increasing in $L$. Therefore, $r^*$ is increasing in $L$ (recall that $C$ is exogenous and therefore constant in this comparative static). Thus, since $P$ is decreasing in $r$ by (1) we have that $p$ is decreasing in $L$.

**Proof of Proposition 3.** Assume first that $P(\bar{B}(C), 0) < \bar{B}(C)$ and assume by contradiction that there exists an equilibrium $(C, r^*, L^*, B^*)$ in which the loan market clears. By Proposition 2, $p(\bar{B}(C); C) = P(\bar{B}(C), 0)$. Since by assumption $P(\bar{B}(C), 0) < \bar{B}(C)$, we therefore have that $p(\bar{B}(C); C) < \bar{B}(C)$. By Corollary 1, $p(L; C)$ is decreasing in $L$ over the region $L > \bar{B}(C)$, so $p(L; C) < L$ for all $L \geq \bar{B}(C)$. Since any equilibrium liquidation value must satisfy $p(L^*; C) = L^*$, this implies that $L^* < \bar{B}(C)$. By Proposition 2(1)(i), however, this implies that the loan market does not clear, contrary to the original assumption.

Suppose now that $P(\bar{B}(C), 0) \geq \bar{B}(C)$ and assume that the level of bank capital is $C$. To show that there exists an equilibrium in which the loan market clears, note again that by Proposition 2, we have that $p(\bar{B}(C); C) = P(\bar{B}(C), 0)$. This implies that $p(\bar{B}(C); C) \geq \bar{B}(C)$. Since $p$ is continuous and bounded from above by $V$ there exists an $L^* \in [\bar{B}(C), V]$ which satisfies $p(L^*; C) = L^*$. By Proposition 1 this $L^*$ is an equilibrium liquidation value. Further, by section (2)(i) of Proposition 2, since $L^* \geq \bar{B}(C)$ the loan market clears.

We therefore have that there exists an equilibrium in which the loan market clears if and only if $P(\bar{B}(C), 0) \geq \bar{B}(C)$.

We made the auxiliary assumption that there is an infinitesimal benefit of borrowing for firms with small borrowing requirement, so that the set of borrowing requirement of actual borrowers in equilibrium is always $[0, B^*]$ for some $B^* \in [0, I]$. Without this assumption, the indirect pricing function is not well-defined, but with some additional assumptions on the pricing function of the liquidated assets, Propositions 3 and 4 are still true, as shown below.
In the general setting, we allow the set of the borrowing requirement of borrowers to be a general measurable subset of $[0, I]$. (In an even more general setting, borrowers with the same borrowing requirement can make different borrowing decisions, but this does not add anything new to the model.) Let $\hat{P} : \mathcal{B}([\prime, \mathcal{I}]) \times \mathbb{C}^+ \to \mathbb{C}$ be the new direct pricing function, where $\mathcal{B}([\prime, \infty])$ is the collection of Borel measurable subsets of $[0, I]$, and $P(B, r)$ is the price of liquidated assets when borrowers with borrowing requirement in $B$ borrowed at Period 0 and the interest rate at Period 0 is $r$. Clearly, $\hat{P}([0, b], r) = P(b, r)$ where $P$ is the pricing function introduced in the paper. To avoid confusion, in the remaining part of this discussion we use $B$ to denote a measurable subset of $[0, I]$ and $b$ a number in $[0, I]$, contrary to the notation used in other parts of the paper. For $B \in \mathcal{B}([\prime, \mathcal{I}])$, let $\mu(B) = \int_B dG(b)!$, the probability measure of $B$.

**Liquidity Pricing Assumption.**
*(i)* $\partial P/\partial b \geq 0$ *for* $r^* = 0$.
*(ii)* $\hat{P}([0, b], 0) \geq \hat{P}(B, 0)$ *for all* $b \in [0, I]$ *and* $B \in \mathcal{B}([\prime, \mathcal{I}])$ *with* $\mu(B) = G(b)$. *(iii)* $\hat{P}(B, r^*)$ *is continuous in* $r^*$ *and* $\partial \hat{P}/\partial r^* \leq 0$ *for all* $B \subset [0, I]$ *with* $\mu(B) \geq G(\bar{B}(0))$.

Assumption (i) is the same as before, and Assumption (iii) is a direct generalization of Assumption (ii) on $P$. Assumption (ii) states that sets of the form $[0, b]$ maximize the price of liquidated assets among all sets with the same probability measure. It is easy to check that all the three assumptions are satisfied by our micro-foundation, as in our micro-foundation

$$\hat{P}(B,r) = \min\left\{V, \max\left\{kV, \frac{1-\gamma}{\gamma}\left((X_1 - (1+r)I)\mu(B) + (1+r)(I - E[b])\right)\right\}\right\},$$

which depends on $B$ only through $\mu(B)$, and is increasing in $\mu(B)$ when $r = 0$, as $X_1 > I$.

**Proof of Proposition 3.***(*under new assumptions) Let $\Pi(r, L) = x_2 + [X_1 - (1+r)L][\gamma + (1-\gamma)L^{-1}V]$. It is easy to see that a firm weakly prefers borrowing if and only if $\Pi(r, L) \geq 0$, and $\bar{r}(L)$ is such that $\Pi(\bar{r}(L), L) = 0$. Clearly, $\Pi$ is continuous in both $r$ and $L$, and $\Pi(0, L) > 0$ for all $L$. We will denote an allocation by $(B, L, r)$ where $B \in \mathcal{B}([\prime, \mathcal{I}])$ is the set of borrowers, $r$ is the interest rate, and $L$ is the liquidation value of assets.
We first prove the "if" part. Since by the assumption $P(\bar{B}(C), r)/(1+r)$ is strictly decreasing in $r$ and approaches zero when $r \to \infty$, there exists $r_1 \in [0, \infty)$ such that $P(\bar{B}(C), r_1)/(1+r_1) = \bar{B}(C)$. Let $L_1 = P(\bar{B}(C), r_1)$.

**Case 1:** $r_1 \leq \bar{r}(L_1)$. Then the allocation $([0, \bar{B}(C)], L_1, r_1)$ is an equilibrium. Indeed, Conditions 2 through 4 for the market equilibrium are satisfied by construction, and Condition 1 is satisfied since all firms weakly prefer borrowing as $r_1 \leq \bar{r}(L_1)$, but only those with borrowing requirement at most $L_1/(1+r_1) = \bar{B}(C)$ are able to borrow.

**Case 2:** $r_1 > \bar{r}(L_1)$. Then by the continuity of $\Pi$ and $P(B, \cdot)$, $\Pi(r, P(\bar{B}(C), r))$ is continuous in $r$. Now $\Pi(r_1, P(\bar{B}(C), r_1)) < 0$ and $\Pi(0, P(\bar{B}(C), 0)) > 0$, so there exists $r_2 \in (0, r_1)$ such that $\Pi(P(\bar{B}(C), r_2), r_2, \bar{B}(C)) = 0$. Let $L_2 = P(\bar{B}(C), r_2)$. Then by Assumption 1, $L_2 \geq L_1$, and thus $L_2/(1+r_2) > \bar{B}(C)$. Therefore, when the interest rate is $r_2$ and the liquidation value of assets is $L_2$, all firms with $B \in [0, L_2/(1+r_2)]$ can borrow, but all firms are indifferent between borrowing and not borrowing, so the allocation $([0, \bar{B}(C)], L_2, r_2)$ is an equilibrium.

Now we prove the "only if" part. In an equilibrium $(B, L, r)$ in which the loan market clears, $\int_B b dG(b) = C + \int_{B^c}(I-b)dG(b)$, so $\mu(B) = \mu([0, \bar{B}(C)]) = G(\bar{B}(C))$. Also, the firm with borrowing requirement $\bar{B}(C)$ must be able to borrow, as borrowers with less borrowing requirement do not have enough mass. Therefore, $\bar{B}(C) \leq L/(1+r) = \hat{P}(B, r)/(1+r) \leq \hat{P}(B, 0) \leq P(\bar{B}(C), 0)$, where we have used the fact that $\mu(B) = G(\bar{B}(C))$ and Assumption (ii) in the last step.
The following corollary is immediate from the proof:

**Corollary A.** *When for a given set of parameters there exists an equilibrium in which the loan market clears, there must exist an equilibrium in which the loan market clears and firms with borrowing requirement in $[0, \bar{B}(C)]$ borrow for the same set of parameters.*

Also, note that in an equilibrium in which the loan market does not clear, the interest rate is zero and all firms strictly prefer borrowing, so the firms who borrow are those who are able to do so; in other words, in such an equilibrium firms with borrowing requirement in $[0, L]$ borrow. The condition for a market equilibrium then implies that $P(L, 0) = L$ if the loan market does not clear. This observation, together with Proposition 3, implies Proposition 4 under the new assumptions.

**Proof of Proposition 4.** The proposition is a direct result of Proposition 2 and Proposition 3. Consider first the case of the conventional equilibrium. Since $P(L,0) > L$ for all $0 < L \leq I$, we know by Proposition 3 that monetary policy is effective at any $C \in [\bar{B}^{-1}(0), \bar{B}^{-1}(I)] = [0, C_{max}]$ where $C_{max} = \int_0^I BdG(B)$.

Next, consider the credit trap equilibrium described in case (ii) of the proposition. Since $P(L,0) \geq L$ for $0 < L \leq L^*$, by Proposition 3 monetary policy is effective over the region $[\bar{B}^{-1}(0), \bar{B}^{-1}(L^*)]$. Further, since $P(L,0) < L$ for $L > L^*$ by Proposition 3 monetary policy is ineffective for any $C > \bar{B}^{-1}(L^*)$. Maximal equilibrium lending is therefore $C^* = \bar{B}^{-1}(L^*)$.

Consider now a level of bank capital $C$ greater than $C^*$. By Proposition 3, the loan market cannot clear, which implies that $r^* = 0$. Since all firms strictly prefer borrowing when $r^* = 0$, $B^* = P(B^*, 0)$, the equilibrium liquidation value of assets. However, the assumption implies that this equality holds only when $B^* = L^*$, so the equilibrium liquidation value is $L^*$ and the aggregate lending is $\int_0^{L^*} BdG(B)$.

Case (iii) of the jump start equilibrium is proved in a similar manner. Since $P(L,0) < L$ for $L_1 < L < L_2$, by Proposition 3 we have that monetary policy is ineffective at any $C \in (\bar{B}^{-1}(L_1), \bar{B}^{-1}(L_2)) = (C_1, C_2)$. Now, since $P(L_2, 0) = L_2$, it is easy to see that when $C = \bar{B}^{-1}(L_2)$, the following allocation is an equilibrium: the marginal borrowing firm is $B^* = L_2$, the interest rate is zero, and the liquidation value of assets is $L_2$. Clearly, in this equilibrium the aggregate lending is $\int_0^{L_2} BdG(B)$.

It remains to show that the equilibrium liquidation value of assets remains $L_1$ for $C \in (\bar{B}^{-1}(L_1), \bar{B}^{-1}(L_2))$ and that the aggregate lending remains $\int_0^{L_1} BdG(B)$ in this case. However, here we can apply the argument used in (ii).

**Proof of Lemma 2.** By assumption a firm liquidates its assets if and only if it is hit by the liquidity shock. Therefore, the total supply of liquidated assets is $\gamma$. Clearly, the price of liquidated assets cannot exceed $V$. When $P(B,r) < V$, all firms with liquidity strictly prefer buying assets, so for the market to clear it must be the case that all market liquidity has been used to buy liquidated assets. In other words, $\gamma P(B,r) = Q(B,r)$ in this case. If $\gamma P(B,r) > Q(B,r)$, then not all the liquidated assets are purchased, which implies that in equilibrium $P(B,r) = 0$, a contradiction. Therefore, the market clearing condition can be written as

$$\gamma P(B,r) \leq Q(B,r), \text{ with equality if } P(B,r) < V,$$

or equivalently,

$$P(B,r) = \min\left\{V, \frac{Q(B,r)}{\gamma}\right\}.$$

**Proof of Lemma 3.** Note that

$$Q(B,r) = \int_0^B [X_1 - (1+r)\tilde{B}]dG(\tilde{B}) + \int_B^I (1+r)(I - \tilde{B})dG(\tilde{B}) = [X_1 - (1+r)I]G(B) + E[A], \quad (2)$$

where $E[A] = I - E[B]$ is the aggregate liquidity at Period 0. Therefore,

$$P(B, r) = \min\left\{V, \frac{Q(B, r)}{\gamma}\right\} = \min\left\{V, \frac{1-\gamma}{\gamma}[X_1 - (1+r)I]G(B) + (1+r)E[A]\right\}.$$

Since $\bar{B}(0)$ satisfies

$$\int_0^{\bar{B}(0)} B dG(B) = \int_{\bar{B}(0)}^{I} (I - B) dG(B),$$

$G(\bar{B}(0)) = E[A]/I$. Therefore, for $B \geq \bar{B}(0)$, $IG(B) - E[A] \geq 0$, and thus $P(B, r)$ is weakly decreasing in $r$. Furthermore, $X_1 > I$, so when $r = 0$, $P(B, r)$ is increasing in $B$.

**Proof of Proposition 5.** We first show that there exists some constant $W_0 > 0$ such that $Q(B, r)/(1-\gamma) \geq W_0$ in equilibrium for all parameter values, where $Q(B, r)$ is the total market liquidity at Period 1. Since the price of liquidated assets is at most $V$, a firm cannot promise to pay more than $V$, and thus $(1+r)\tilde{B} \leq V$ if a firm with borrowing requirement $\tilde{B}$ borrows in equilibrium. Therefore,

$$Q(B, r) = (1-\gamma)\int_0^B [X_1 - (1+r)\tilde{B}] dG(\tilde{B}) + (1+r)\int_B^I (I - \tilde{B}) dG(\tilde{B}) \geq (1-\gamma)\int_0^I w(\tilde{B}) dG(\tilde{B}),$$

where

$$w(\tilde{B}) \quad = \begin{cases} X_1 - V, & \text{if } \tilde{B} \leq B; \\ I - \tilde{B}, & \text{otherwise.} \end{cases}$$

Now $w(\tilde{B}) \geq 0$ for all $\tilde{B} \in [0, I]$, and also $w(\tilde{B}) \geq \min\left\{X_1 - V, \frac{I}{2}\right\}$ for $\tilde{B} \leq I/2$. Put $W_0 = G(I/2)\min\left\{X_1 - V, \frac{I}{2}\right\}$. Then

$$Q(B, r) \geq (1-\gamma)\int_0^{\frac{I}{2}} q(\tilde{B}) dG(\tilde{B}) \geq (1-\gamma)W_0.$$

Now choose $\bar{\gamma}$ so that $(1-\bar{\gamma})W_0/\bar{\gamma} = V$. Then in equilibrium $Q(B, r)/\gamma \geq V$ for all $\gamma \leq \bar{\gamma}$, and thus $P(B, r) = V$. Clearly, when $I < V$ this also means that we are in a conventional equilibrium. (Note: when a general set of borrowers, which can be any measurable subset of $[0, I]$, is allowed in equilibrium, the proof goes essentially unchanged, except that now $w(\tilde{B}) = (X_1 - V - I + \tilde{B})1_{\mathcal{B}} + I - \tilde{B}$ where $\mathcal{B}$ is the set of borrowers in equilibrium.)

**Proof of Proposition 6.** We first show that $\bar{L}$ can be attained. An equilibrium is characterized by $(B, L, r)$, where $B$ is the borrowing requirement of the marginal borrower, $L$ is the liquidation value of assets, and $r$ is the interest rate. Since $L \leq V$ in all equilibrium, there exists a sequence of equilibriums $(B_n, L_n, r_n)$ for bank capital $C_n$ such that $L_n \to \sup_C L^*$. Since $B_n \in [0, I]$, $L_n \in [0, V]$, and $r_n \in [0, \bar{r}(V)]$ for all $n$, one can choose a subsequence $(B_{n_k}, L_{n_k}, r_{n_k})$ such that $(B_{n_k}, L_{n_k}, r_{n_k})$ converges to some $(\hat{B}, \hat{L}, \hat{r})$ as $k \to \infty$. Since all equilibrium conditions are continuous, $(\hat{B}, \hat{L}, \hat{r})$ is an equilibrium for some $\hat{C}$. Clearly, $\hat{L} = \sup_C L^*$.Therefore, the supremum of $L^*$ can be attained, and thus $\bar{L} = \max_C\{L^*\}$ is well-defined. The following lemma will be also be useful later:

**Lemma A1.** *If $P(I,0) < I$, then $\bar{L} = \max\{L \in [0,I] : P(L,0) \geq L\}$. Furthermore, the maximum aggregate lending in equilibrium is $\int_0^{\bar{L}} BdG(B)$. (As before, the maximum is taken over all stances of monetary policy.)*

*Proof.* Let $\hat{L} = \sup\{L : P(L,0) \geq L\}$. Then by the continuity of $P$, $\hat{L}$ can be attained. Since $P(I,0) < I$, $\hat{L} < I$, and by the continuity of $P$ again $P(\hat{L},0) = \hat{L}$. This implies that the following allocation is an equilibrium: firms with borrowing requirement in $[0,\hat{L}]$ borrows, the interest rate is zero, and the liquidation value of assets is $\hat{L}$. Therefore, $\bar{L} \geq \hat{L}$.
Let $(B^*, \bar{L}, r^*)$ be an equilibrium that attains the maximum liquidation value of assets. If $r^* > 0$, then $B^* = \bar{B}(C)$ for some $C \in [0, C_{max}]$. By Proposition 3 $P(B^*,0) \geq B^*$. Moreover, $P(B^*,r^*) \leq P(B^*,0)$. If $r^* = 0$, then it must be the case that all firms strictly prefer borrowing, and the definition of market equilibrium implies that $P(B^*,0) = \bar{L} = B^*$. Therefore, we always have $P(B^*,r^*) \leq P(B^*,0)$ and $P(B^*,0) \geq B^*$. By the construction of $\hat{L}$, $B^* \leq \hat{L}$. Now by the monotonicity assumptions on $P$,

$$\bar{L} = P(B^*,r) \leq P(B^*,0) \leq P(\hat{L},0) = \hat{L}.$$

Therefore, $\bar{L} = \hat{L}$.
Now we show that the maximum aggregate lending is $\int_0^{\bar{L}} BdG(B)$. We have seen that $(\bar{L},\bar{L},0)$ is an equilibrium. (i.e. the marginal borrower and the liquidation value of assets are both $\bar{L}$, and the interest rate is zero.) In this equilibrium the aggregate lending is $\int_0^{\bar{L}} BdG(B)$. Suppose that there exists some equilibrium $(B', L', r')$ in which the aggregate lending is more than $\int_0^{\bar{L}} BdG(B)$. Then since only borrowers with borrowing requirement at most $L'/(1+r')$ can borrow, $\int_0^{L'/(1+r')} BdG(B) > \int_0^{\bar{L}} BdG(B)$, implying that $L' > (1+r')\bar{L} \geq \bar{L}$, contradicting the maximality of $\bar{L}$. Therefore, the maximum aggregate lending in equilibrium is $\int_0^{\bar{L}} BdG(B)$. □

Now we return to the proof of Proposition 5. Note that by Eq. (2) in any equilibrium $(B,L,r)$ the market liquidity at Period 1 is

$$Q(B,r) = (1-\gamma)\{[X_1 - (1+r)I]G(B) + (1+r)E[A]\},$$

where $E[A]$ is the expectation of firms' liquidity at Period 0. For $B < \bar{B}(0)$, $r$ must be zero as otherwise the clearing of the loan market would imply a negative $C$, and for $B \geq \bar{B}(0)$, $Q(B,r)$ is weakly decreasing in $r$ as in that case $IG(B) - E[A] \geq IG(\bar{B}(0)) - E[A] = 0$. Therefore,

$$Q(B,r) \leq Q(B,0) \leq (1-\gamma)(X_1 - I + E[A]).$$

Put $\gamma_1 = 1 - V/(X_1 - I + E[A])$. Then

$$\frac{Q(B,r)}{\gamma} \leq \frac{1-\gamma}{\gamma}(X_1 - I + E[A]) < V,$$

when $\gamma > \gamma_1$. This implies that $L = \min\{V, Q(B,r)/\gamma\} < V$, for all equilibrium $(B,L,r)$. Consequently $\bar{L} < V$. Note also that when $\gamma > \gamma_1$,

$$P(I,0) = \frac{1-\gamma}{\gamma}(X_1 - I + E[A]).$$

Put

$$\gamma_2 = \frac{X_1 - I + E[A]}{X_1 + E[A]}.$$

Then $P(I,0) < I$ for $\gamma > \max\{\gamma_1, \gamma_2\}$. Put

$$\bar{\gamma} = \max\{\gamma_1, \gamma_2\}. \tag{3}$$

In what follows, assume that $\gamma > \bar{\gamma}$.

(i) We have seen that $L < V$ in all equilibrium, and $\bar{L}$ can be attained, so $\bar{L} < V$. By Lemma A1, $P(L,0) < L$ for all $L \in (\bar{L}, I]$. That the monetary policy is ineffective beyond $\bar{B}^{-1}(\bar{L})$ follows directly from Proposition 3.

(ii) In this part, we write $\bar{L}(\gamma)$ to emphasize the dependence of $\bar{L}$ on $\gamma$. Fixing $\gamma', \gamma\text{''} \in (\bar{\gamma}, 1)$ with $\gamma'' < \gamma'$, we show that $\bar{L}(\gamma'') > \bar{L}(\gamma')$. Let $(B^*, \bar{L}(\gamma'), r^*)$ be an equilibrium that attains the maximum liquidation value of assets when the probability of the liquidity shock is $\gamma'$. Then the definition of an equilibrium implies that $r^*$ and $\bar{L}(\gamma')$ solves the following system with $\gamma = \gamma'$:

$$\begin{cases} r = \min\left\{\bar{r}(L;\gamma), \frac{L}{B^*} - 1\right\}; \\ L = P(B^*, r; \gamma), \end{cases}$$

where we have emphasized the dependence of $\bar{r}$ and $P$ on $\gamma$. It is easy to see that $\bar{r}$ is strictly increasing in $\gamma$ and $P$ is strictly decreasing in $\gamma$. There are two cases:

**Case 1:** $B^* < \bar{B}(0)$. Then $r^* = 0$ and $\bar{L}(\gamma') = B^* = P(B^*, 0; \gamma')$. Since $P$ is strictly decreasing in $\gamma$, $P(B^*, 0; \gamma'') > B^*$. Let $B' = \min\{\bar{B}(0), \sup\{\tilde{B} : P(\tilde{B}, 0; \gamma'') > \tilde{B}\}\}$. Then $B' > B^*$ and $P(B', 0; \gamma'') \geq B'$ with equality when $B' < \bar{B}(0)$. If $B' = \bar{B}(0)$, by Proposition 3 there exists an equilibrium $(B', L', r')$ when $\gamma = \gamma''$. Then

$$\bar{L}(\gamma'') \geq L' \geq (1 + r')B' \geq \bar{B}(0) > B^* = \bar{L}(\gamma').$$

If $B' < \bar{B}(0)$, then $(B', B', 0)$ is an equilibrium, and thus $\bar{L}(\gamma'') \geq B' > B^* = \bar{L}(\gamma')$.

**Case 2:** $B^* \geq \bar{B}(0)$. Then $P(B^*, 0; \gamma'') > P(B^*, 0; \gamma') \geq B^*$, and by Proposition 3, there exists an equilibrium $(B^*, L', r')$ when $\gamma = \gamma''$. Suppose $L' \leq \bar{L}(\gamma')$. Then

$$r' = \min\left\{\bar{r}(L'; \gamma''), \frac{L'}{B^*} - 1\right\} \leq \min\left\{\bar{r}(\bar{L}(\gamma'); \gamma'), \frac{\bar{L}(\gamma')}{B^*} - 1\right\} = r^*,$$

and thus

$$L' = P(B^*, r'; \gamma'') > P(B^*, r^*; \gamma') = \bar{L}(\gamma'),$$

a contradiction. Therefore, $L' > \bar{L}(\gamma')$ and thus $\bar{L}(\gamma'') > \bar{L}(\gamma')$.

(iii) We have seen that in all equilibrium $(B, L, r)$,

$$L = \frac{Q(B,r)}{\gamma} \leq \frac{Q(B,0)}{\gamma} \leq \frac{1-\gamma}{\gamma}(X_1 - I + E[A]),$$

so

$$\bar{L} \leq \frac{1-\gamma}{\gamma}(X_1 - I + E[A]),$$

but the right hand side approaches zero as $\gamma$ approaches one.

**Proof of Proposition 7.** The proofs of Propositions 7 and 8 rely on the following lemma.

**Lemma A2.** Given a distribution $G$ and $(X_1, V, I)$ with $\bar{B}(0) < V$, there exists $\gamma_1 < \gamma_2$ such that quantitative easing will be successful in increasing lending for $\gamma \in [\gamma_1, \gamma_2]$ if

$$\frac{\min\{V, I\}}{(X_1 - I)G(\min\{V, I\}) + E[A]} < \frac{I\bar{B}(0)}{X_1 E[A]}. \tag{4}$$

*Proof.* Let $f(L) = P(L, 0) - L$. Clearly, $f(L)$ is continuous in $L$. Clearly, quantitative easing will be successful if $f(\bar{B}(0)) < 0$ and $f(L) > 0$ for some $L \in (\bar{B}(0), \min\{V, I\}]$, by Proposition 3. Note that

$$f(\bar{B}(0)) = \min\left\{V, \frac{1-\gamma}{\gamma}[(X_1 - I)G(\bar{B}(0)) + E[A]]\right\} - \bar{B}(0).$$

Let

$$\alpha_1 = \frac{\bar{B}(0)}{(X_1 - I)G(\bar{B}(0)) + E[A]}.$$

Then $f(\bar{B}(0)) < \bar{B}(0)$ if $(1-\gamma)/\gamma < \alpha_1$. Using the definition of $\bar{B}(0)$, $G(\bar{B}(0)) = E[A]/I$, so $\alpha_1 = (\bar{B}(0)I)/(X_1 E[A])$.
Let

$$\alpha_2 = \frac{\min\{V, I\}}{(X_1 - I)G(\min\{V, I\}) + E[A]}.$$

Then by the continuity of $G$, when $(1-\gamma)/\gamma > \alpha_2$, one can find $L \in (\bar{B}(0), \min\{V, I\})$ such that

$$\frac{1-\gamma}{\gamma}[(X_1 - I)G(L) + E[A]] > \min\{V, I\},$$

which implies that

$$f(L) = \min\left\{V, \frac{1-\gamma}{\gamma}[(X_1 - I)G(L) + E[A]]\right\} - L > \min\left\{V, \frac{1-\gamma}{\gamma}[(X_1 - I)G(L) + E[A]]\right\} - \min\{V, I\} \geq 0.$$

Therefore, if $\alpha_2 < \alpha_1$, which is exactly Eq. (4), then quantitative easing will be successful for $\gamma \in (1/(1+\alpha_1), 1/(1+\alpha_2))$. □

Now we return to the proof of Proposition 7. Note that when $I < V$, the left hand side of Eq. (4) becomes $I/(X_1 - I + E[A])$, and thus Eq. (4) is equivalent to the following condition:

$$(\bar{B}(0) - E[A])X_1 > \bar{B}(0)(I - E[A]). \tag{5}$$

Let $A_0 = (X_1 - I)\delta/X_1$ and $\epsilon = (X_1 - I)/(X_1 I)$. If $\bar{B}(0) \geq \delta$, then Eq. (5) holds because

$$(\bar{B}(0) - E[A])/\bar{B}(0) > (\bar{B}(0) - A_0)/\bar{B}(0) \geq 1 - \frac{X_1 - I}{X_1} > (I - E[A])/X_1.$$

Now assume that $\bar{B}(0) < \delta$. Note that since $E[A] = IG(\bar{B}(0))$, we have

$$[\bar{B}(0) - E[A]]X_1 = [\bar{B}(0) - IG(\bar{B}(0))]X_1 > X_1\bar{B}(0)(1 - I\epsilon) = I\bar{B}(0) > \bar{B}(0)(I - E[A]),$$

where we have used the fact that $G'(L) = g(L) < \epsilon$ for all $L \in (0, \delta)$ in the second step. Therefore, Eq. (5) is satisfied in this case too.

**Proof of Proposition 8.** Again, under the assumption that $I < V$, Eq. (4) is equivalent to Eq. (5). Since $G$ is strictly convex,

$$E[A] = IG(\bar{B}(0)) < I\left[\frac{I - \bar{B}(0)}{I} G(0) + \frac{\bar{B}(0)}{I} G(I)\right] = \bar{B}(0).$$

Let $\underline{X_1} = \bar{B}(0)(I - E[A])/(\bar{B}(0) - E[A])$. Then $\underline{X_1} > 0$ and Eq. (5) holds when $X_1 > \underline{X_1}$.

**Proof of Proposition 9.** Since $X_1 < I/(1 - \gamma)$,

$$\frac{1-\gamma}{\gamma}(X_1 - I) < I.$$

Therefore, there exists $A_0 > 0$ and $C^* < C_{max} \equiv \bar{B}^{-1}(I)$ such that

$$\frac{1-\gamma}{\gamma}(X_1 - I) + A_0 < \bar{B}(C^*).$$

Then for any distribution $G$ with $E[A] \leq A_0$ and any $C \in [C^*, C_{\max}]$, we have

$$P(\bar{B}(C), 0) = \min\left\{V, \frac{1-\gamma}{\gamma}(X_1 - I)G(\bar{B}(C)) + E[A]\right\} < \frac{1-\gamma}{\gamma}(X_1 - I) + A_0 < \bar{B}(C^*) \leq \bar{B}(C).$$

Therefore, by Proposition 3, monetary policy is ineffective beyond $C^*$.

**Proof of Proposition 10.** To emphasize the dependence of $P$, $E[A]$ and $\bar{B}$ on the distribution $G$, we will write $P(B, r; G)$, $E_G[A]$ and $\bar{B}(C; G)$, respectively. By Proposition 3 and the assumption, $P(I, 0; G_1) < I$. By Lemma A2, $\bar{L}_1 = \max\{L \in [0, I] : P(L, 0; G_1) \geq L\}$. Consequently, $P(L, 0; G_1) < L$ for all $L \in (\bar{L}_1, I]$.
Since $G_2$ stochastically dominates $G_1$, $E_{G_2}[A] < E_{G_1}[A]$, and $G_2(L) \leq G_1(L)$ for all $L \in [0, I]$. Therefore, for all $L \in [0, I]$,

$$\begin{aligned}
& P(L, 0; G_2) \\
= \;& \min\left\{V, \frac{1-\gamma}{\gamma}((X_1 - I)G_2(L) + E_{G_2}[A])\right\} \\
\leq \;& \min\left\{V, \frac{1-\gamma}{\gamma}((X_1 - I)G_1(L) + E_{G_1}[A])\right\} \\
= \;& P(L, 0; G_1).
\end{aligned}$$

In particular, $P(L, 0; G_2) \leq P(L, 0; G_1) < L$ for all $L \in (L_1^*, I]$. Therefore, Lemma A1 applies, and $\bar{L}_2 = \max\{L \in [0, I] : P(L, 0; G_2) \geq L\} \leq \bar{L}_1$. The result on the aggregate lending also follows from Lemma A1.